

Book Review

Twelve Questions on Mathematics Teaching

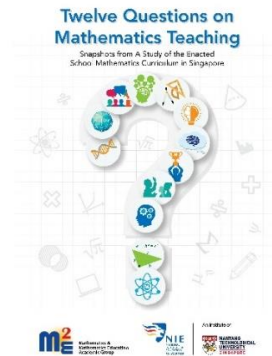
Snapshots from a Study of the Enacted School Mathematics Curriculum in Singapore

Kaur, Toh TL, Lee, Leong, Cheng, Ng, Yeo KKJ, Yeo BWJ, Wong, Tong, Toh WYK and Safii (2019)

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This paper reviews the book *Twelve Questions on Mathematics Teaching* by Kaur et al. The book reports on the results of a research project, *A Study of the Enacted School Mathematics Curriculum*, conducted by a unique blend of mathematics educators and mathematicians from the Mathematics and Mathematics Education Academic Group of the National Institute of Education in Singapore. The book presents how 30 experienced and competent mathematics teachers across all four secondary mathematics courses (Integrated Programme, Express, Normal [Academic], Normal [Technical]) in Singapore enact the Singapore Mathematics Curriculum. Then, it compares these results to 677 mathematics teachers surveyed across Singapore. Implications for effective teaching are discussed. Given Singapore's positive results in large-scale international studies, such as TIMSS and PISA, the book provides a fascinating and valuable opportunity for educators within and beyond Singapore to observe the inner workings of Singapore mathematics classrooms.

The book addresses 12 important questions about Singapore mathematics teachers' adopted pedagogies and the instructional materials they use. It provides a range of examples of how experienced and competent mathematics teachers introduce concepts, develop skill proficiency, emphasise mathematical processes, develop metacognitive strategies, and imbue desired attitudes for their students. Furthermore, the book explains how experienced and competent mathematics teachers use instructional materials to help students make connections, to stimulate their thinking, and to help them to develop and apply formulae. These examples and their analysis are a useful asset for future and present mathematics teachers and educators to reflect upon their own teaching practices and to refine them to deliver teaching that is more effective. Despite the common belief that Singapore mathematics teachers, along with teachers from several other Asian nations, inherently adopt "drill and practice" teaching strategies,

Twelve Questions on Mathematics Teaching challenges this narrative by demonstrating over 12 chapters that things are much more complex than what they seem.

Brief synopsis of the 12 chapters are as follows.

1. *What is the instructional core that guides mathematics teaching in the classrooms of Singapore secondary schools?*

The opening chapter of the book discusses the three instructional core components, namely D – teacher develops concepts or demonstrates skills, S – teacher sets student work, and R – teacher reviews students' work. It explains how experienced and competent teachers undergo several cycles and combinations of D-S-R within one lesson, which is also reflective of teachers in general across Singapore. Instead of the traditional Introduction, Development, Consolidation, and Conclusion instructional phases of a lesson, this chapter explains how lessons can be dissected into D-, S-, and R-components to clarify how teachers achieve micro-instructional objectives. The chapter also details the unique way that Singapore teachers use students' work from the S-component as a basis for whole class discussions in the R-component that illuminates how Singapore mathematics lessons are indeed teacher-led yet student-centred.

2. *Which models of instruction guide mathematics teaching in the classrooms of Singapore secondary schools?*

This chapter explores the models of instruction that guide teachers' instructional practice. For the 30 experienced and competent mathematics teachers, three models were found. These were *traditional instruction* (TI) – typically associated with teacher-centred drill and practice, *direct instruction* (DI) – involving teacher-centred guidance with checks for mastery, and *teaching for understanding* (TfU) – involving teachers' facilitation, monitoring and regulating of student-centred learning. The chapter presents how experienced and competent teachers use these models and the role of the student as observed in lessons. It also includes a list of ways that teachers in general across all four course enact in their teaching. The potential for a fourth model, *Co-Regulated Learning Strategies*, is discussed and the chapter argues that teachers need to make greater efforts to shift from being disseminators of knowledge to facilitators of knowledge construction.

3. *What kinds of math talk are enacted in the classrooms of Singapore secondary schools?*

This chapter examines the use of meaningful math talk and distinguishes between *univocal* and *dialogic* talk. Three teaching episodes of one teacher, Mr R, and his students demonstrate how the different forms of *teaching* and *learning talk* contribute to students' understanding of a concept. For teachers in general, the chapter compares the frequency of use of the different forms of teaching and learning talk, distinguishing those that are more basic learning talk (e.g., narrating and explaining) and more complex learning talk (e.g., exploring, arguing, discussing). The chapter acknowledges that the reality of classrooms contributes to the predominance of univocal talk but presents the implications of engaging students in more dialogic talk.

4. *How do mathematics teachers in Singapore secondary schools introduce concepts to students or engage them in constructing concepts?*

The chapter deals with the use of representations and their translation for students' concept construction. It illustrates three ways that experienced and competent teachers introduce

concepts and use different forms of representations. Namely, they explain a problem then ask questions, they guide students to discover a formula, and they engage students in an inductive activity to discover a formula. Notably, the chapter identifies slightly contrasting results for Singapore teachers in general. The chapter argues for the importance of balancing conceptual understanding and skill fluency and provides suggestions for how this can be achieved.

5. How do mathematics teachers in Singapore secondary schools engage students in developing proficiencies in mathematics skills?

This chapter explicates how teachers structure their lessons and how they use worked examples to develop students' skill proficiency. It provides insights into the variety of ways that teachers may incorporate cycles of *Introduction, Development, Consolidation* and *Conclusion* within one lesson or across lessons to promote relational understanding for developing skill proficiency. Worked examples were a key approach used by Singapore teachers in general and illustrations of experienced and competent teachers' use of the "I do, We do, You do" method are given. The chapter concludes with suggestions for how to plan well-structured lessons that will be more effective for developing students' skill proficiency meaningfully.

6. How do mathematics teachers in Singapore secondary schools emphasise mathematical processes?

The chapter evaluates how teachers build students' mathematical reasoning and communication, thinking skills and heuristics, and mathematical applications in the real world. Episodes of how experienced and competent teachers used students' work to develop these mathematical processes are provided. These episodes demonstrate how teachers challenge students' thinking, strategically prompt and guide students, and facilitate connections between real world and other mathematical concepts. The chapter discusses the similar results found for Singapore mathematics teachers in general and draws attention to specific methods for readers to incorporate into future lessons. Suggestions are given for how to provide more opportunities for students to articulate their mathematical processes.

7. How do mathematics teachers in Singapore secondary schools facilitate the development of metacognitive strategies amongst their students?

This chapter addresses how teachers help students to develop their metacognitive strategies, namely their *awareness, monitoring* and *regulation*. Three episodes of experienced and competent teachers are provided to demonstrate how they encourage students to compare solving methods, to reflect upon their learning and prior knowledge, and to check and justify the reasonableness of their answers. While checking for understanding of the problem prior to solving was less dominant in experienced and competent teachers, it was a key approach used by teachers in general. Suggestions and justifications for the use of these approaches is given for teachers to consider that address the three components of metacognition.

8. How do mathematics teachers in Singapore secondary schools imbue desired attitudes for the learning of mathematics amongst their students?

The chapter explores teachers' enactment of attitudes, the final component of the Singapore Mathematics Curriculum. It illustrates the different strategies that experienced and competent teachers use to build students' confidence, encourage them to persevere, appreciate the relevance of mathematics, and to appeal to their interests. These strategies are also reflective of the *LOVE Mathematics framework* which presents *variety, opportunities, and linkages* as key aspects to include in lessons to increase students' engagement. Results of the survey among

Singapore mathematics teachers in general indicate that teachers frequently addressed building confidence, perseverance and appreciation in lessons. An interesting part to note are the ways that Singapore teachers have attempted to foster students' interests in mathematics. The chapter concludes with advice that teachers should consider making lessons more interesting to students that would help to motivate them to learn.

9. How do mathematics teachers in Singapore secondary schools use instructional materials to help students make connections?

This chapter is the first in the book to address the unique ways that Singapore mathematics teachers use instructional materials. It centres on the importance of making connections and discusses how experienced and competent mathematics teachers use different representations, develop earlier concepts, and reinforce established conceptual links. Useful examples of how teachers incorporate algebraic, graphical, pictorial and tabular representations into a single activity at various year levels for various topic are given. Readers may find these examples useful to guide the design of their own instructional materials to build and strengthen students' connections between mathematical concepts and the real world.

10. How do mathematics teachers in Singapore secondary schools use instructional materials to stimulate students' thinking?

This chapter discusses how teachers can design activities to stimulate students' thinking beyond procedural operations. Amongst the 30 experienced and competent mathematics teachers, two strategies that teachers used were to include challenging problems and to facilitate comparisons between solving methods. The chapter also reports on the frequency of use of these strategies by Singapore mathematics teachers in general. Examples of challenging problems show how incorporating additional conditions can increase their difficulty to stimulate students' thinking. Furthermore, the examples of activities demonstrate how teachers might extend students' thinking by asking them to justify their solving methods.

11. How do mathematics teachers in Singapore secondary schools use instructional materials to help students develop and apply formulae?

The chapter presents the final strategy Singapore mathematics teachers use to address students' development and application of formulae. It presents three ways that experienced and competent teachers help students gain fluency with formulae by beginning with easier items, using a variety of examples, and developing from students' previously learnt formulae. The chapter includes six examples relating to differentiation, trigonometry ratios, linear equations, quadratic equations, the quadratic formula, and area of triangles. It demonstrates how to include a progression in difficulty that can encourage students to generalise the use of a formula, how slight variations in subsequent problems can help to illuminate important aspects of formulae, and how drawing connections with a previously learnt formula can assist students to develop an understanding of new formulae.

12. How mathematically powerful are the classrooms in Singapore secondary schools?

The final chapter of the book applies Alan Schoenfeld's *Teaching for Robust Understanding of Mathematics* (TRU Math) framework to analyse the power of Singapore mathematics classrooms based on the findings discussed from the prior chapters. The framework is presented and five components are described: *the mathematics*; *cognitive demand*; *access to mathematical content*; *agency, authority and identity*; and *uses of assessment*, each with three levels. The discussion in this chapter explains how Singapore mathematics classrooms are

mathematically powerful and the activities that teachers use go beyond the first level of the TRU Math framework. The chapter concludes with final recommendations for how Singapore mathematics teachers may advance all five components in their classrooms to make them more mathematically powerful.

If one wishes to understand the effective practices of Singapore mathematics teachers, *Twelve Questions on Mathematics Teaching* is an excellent place to start. The book provides a unique opportunity for readers to step into the minds of 30 experienced and competent mathematics teachers to understand their pedagogies and use of instructional materials. The book demonstrates that what goes on in Singapore classrooms is a far cry from the typically assumed drill and practice teaching methods. It illustrates the complex and deliberate ways that teachers enact the Singapore Mathematics Curriculum and provides useful examples of activities that readers may adapt and incorporate into their own teaching. Furthermore, *Twelve Questions on Mathematics Teaching* inherently encourages readers to reflect on their own teaching practices and provides suggestions for how to improve the effectiveness of their teaching.

Reviewer

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