Characterizing and Supporting Hybrid Learning Scenarios to Foster Students' Development of Mathematical Concepts and Problem-Solving Competencies

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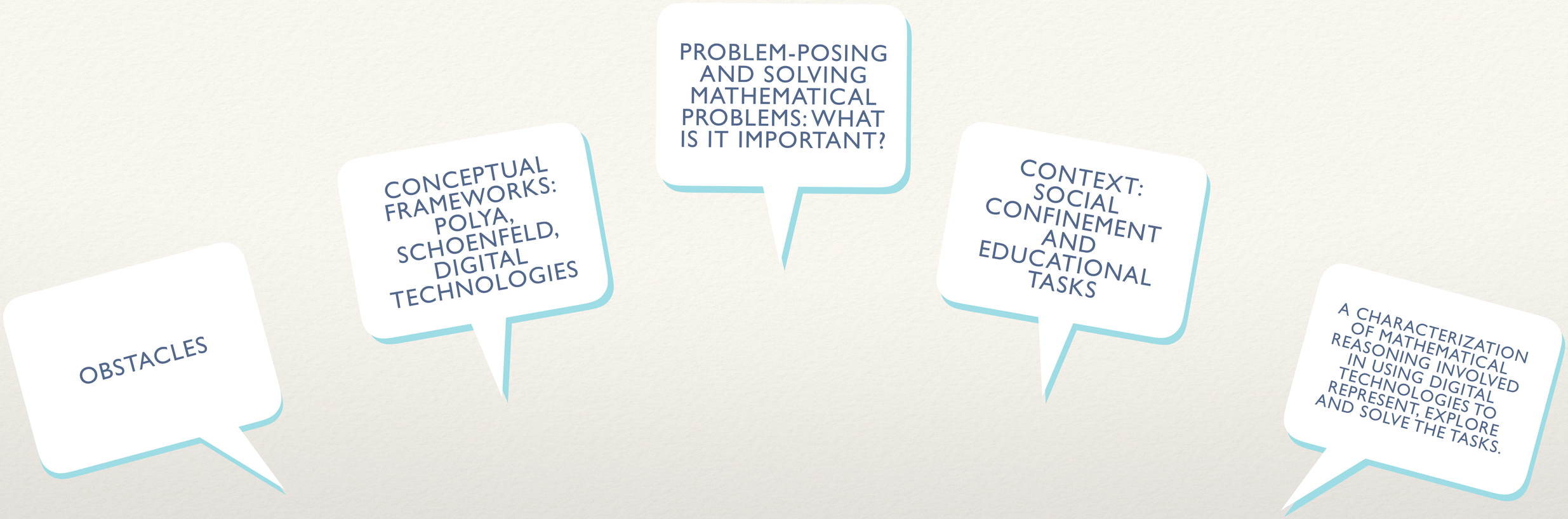

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Context: What have we learned from school work during the social confinement?

1. Quick transition to online or remote teaching/leaning scenarios, the emergence of a hybrid approach
2. Teachers and students encountered obstacles such as uneven internet access
3. The absence of proper materials to support instruction
4. The lack of tools to monitor and assess students' learning and problem-solving competencies
5. Teachers/students difficulties to get essential resources and professional support to frame teaching practices in current learning spaces

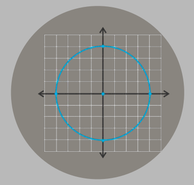
Tools, mathematics and mathematics education

Mathematics developments and results can be traced and explained in terms of what tools individuals or groups used to formulate and solve problems

Tools amplify human cognition (memory, computation, representation, etc.)

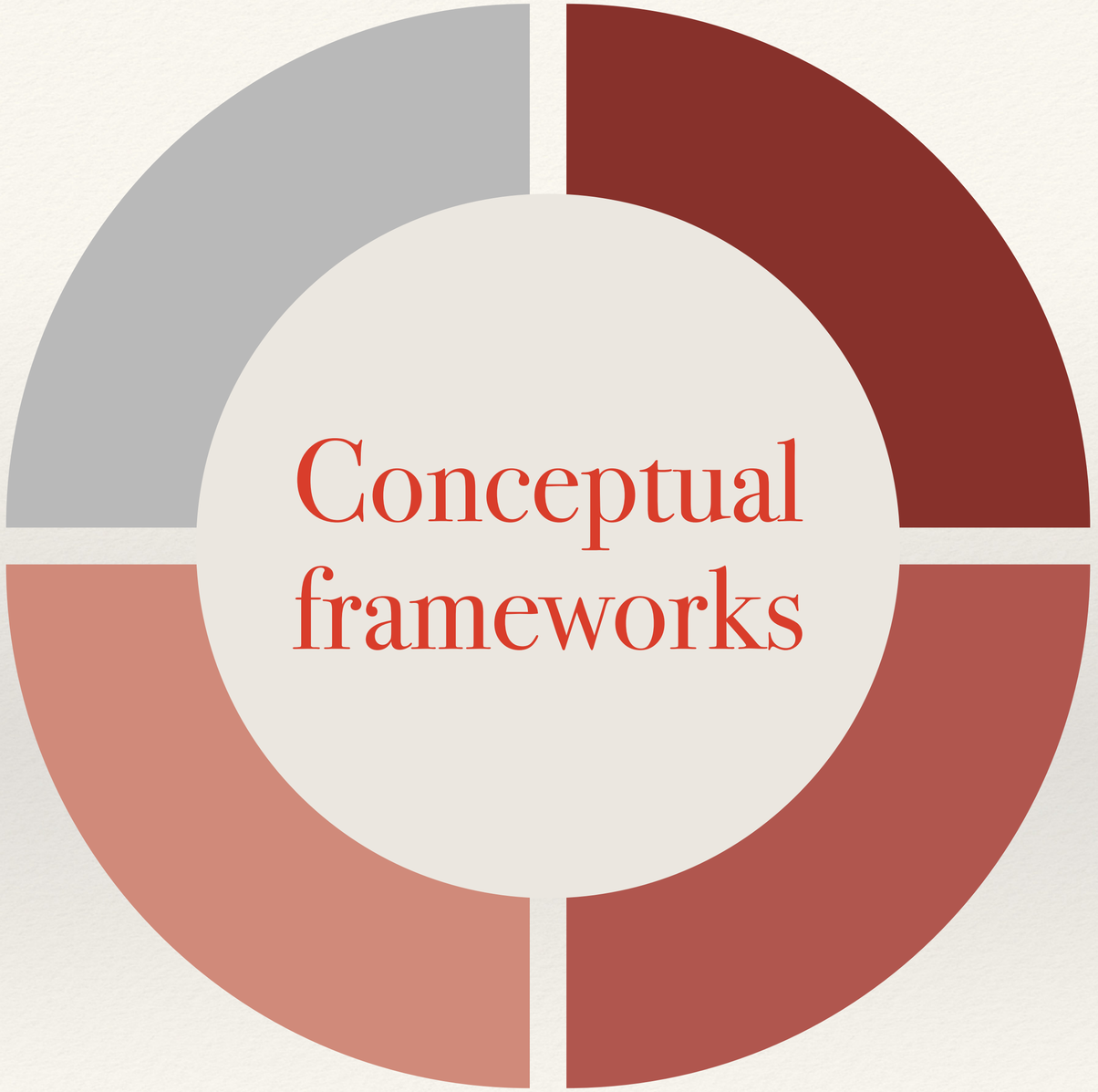
Babylonian mathematicians (1830-1531) used clay tablets to register problems, methods, and results in arithmetic, geometry and equations

Greek mathematicians relied on the straightedge and compass to work on geometry problems. Euclid (325 BC) introduced the axiomatic method to support and validate mathematical results

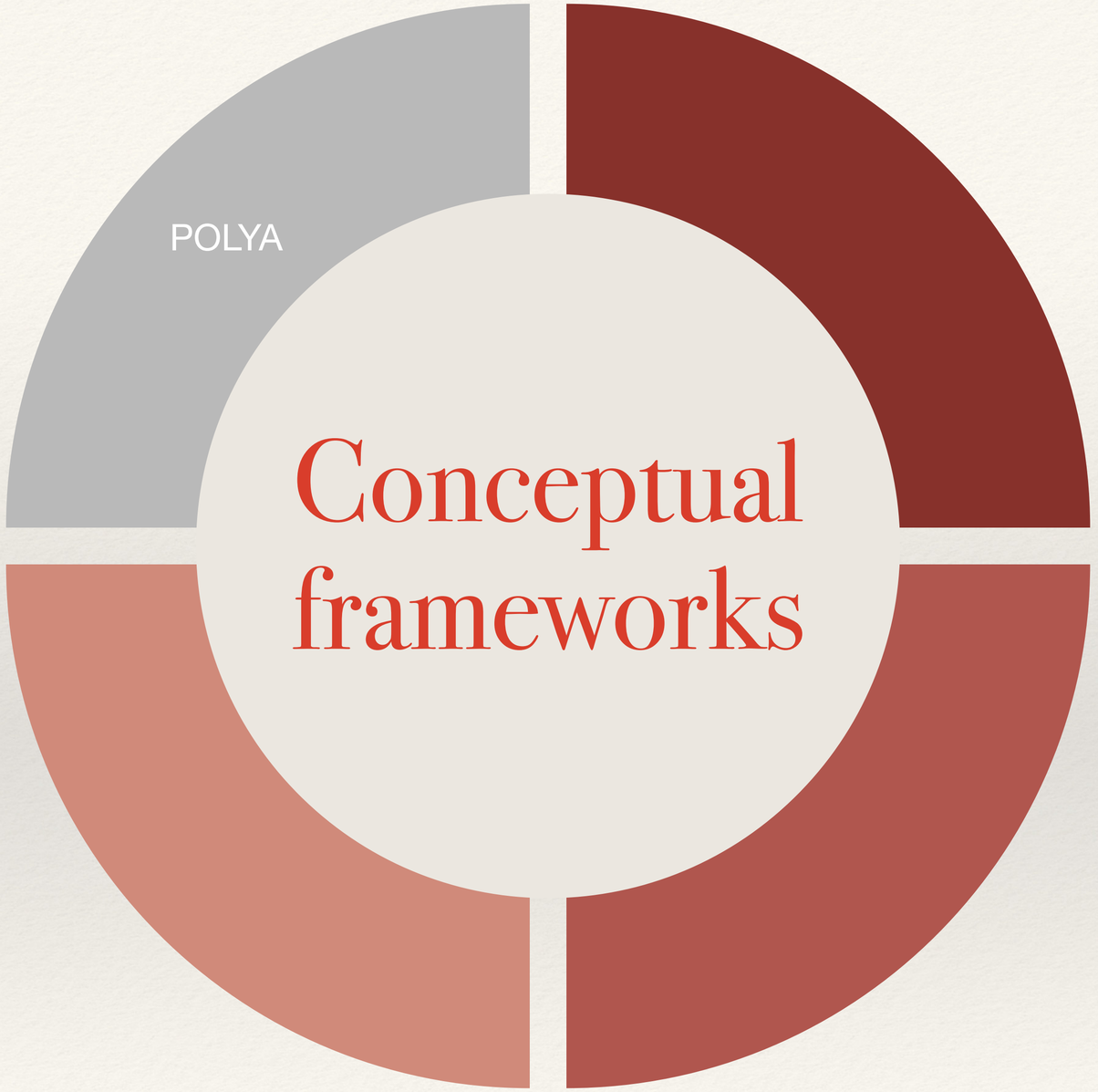


Descartes (1596-1650) introduced the coordinate systems to the study of geometry (analytic geometry)

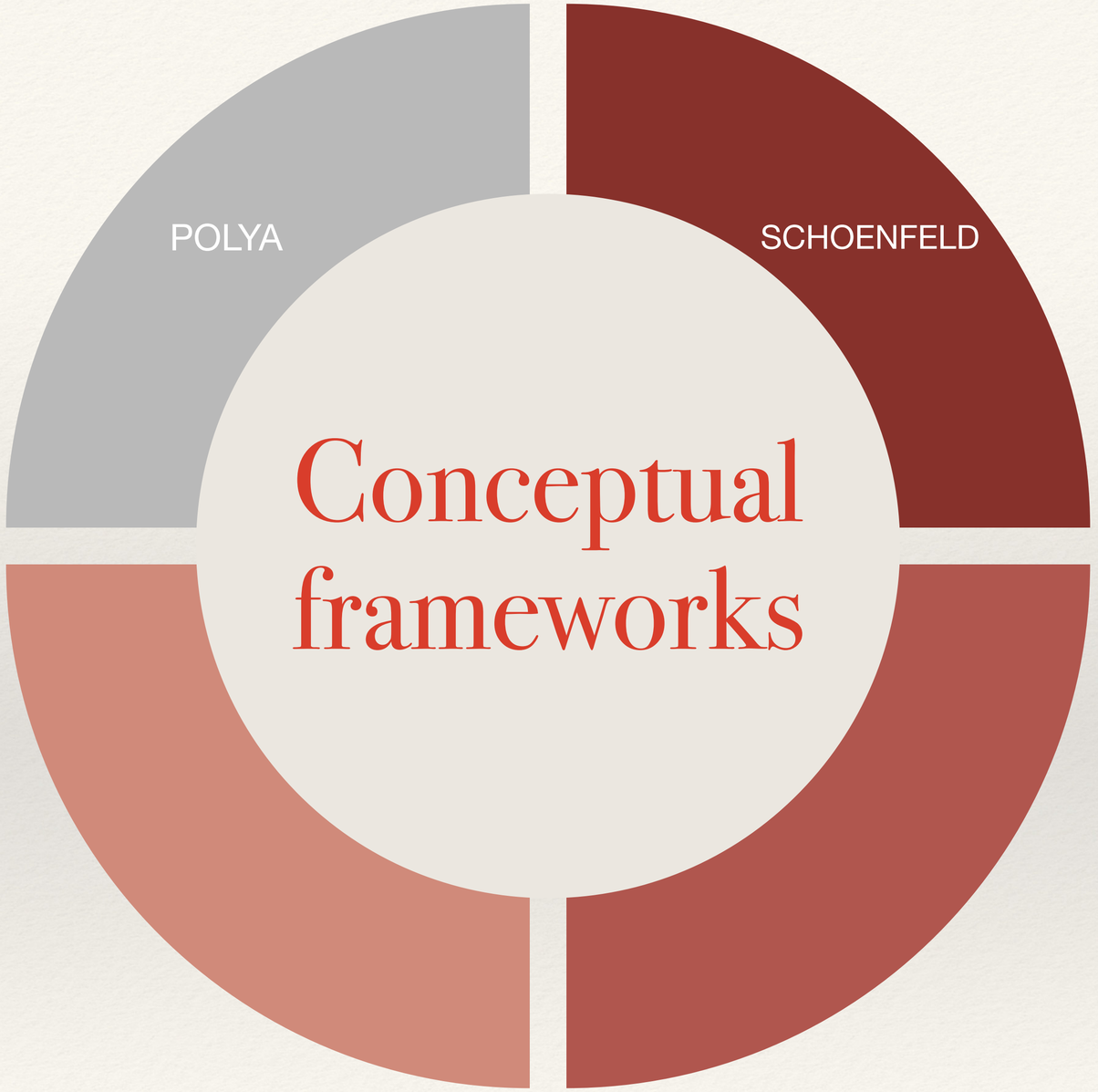
Currently, digital technologies are shaping the way we work on mathematical problems


Conceptual
frameworks

Conceptual frameworks

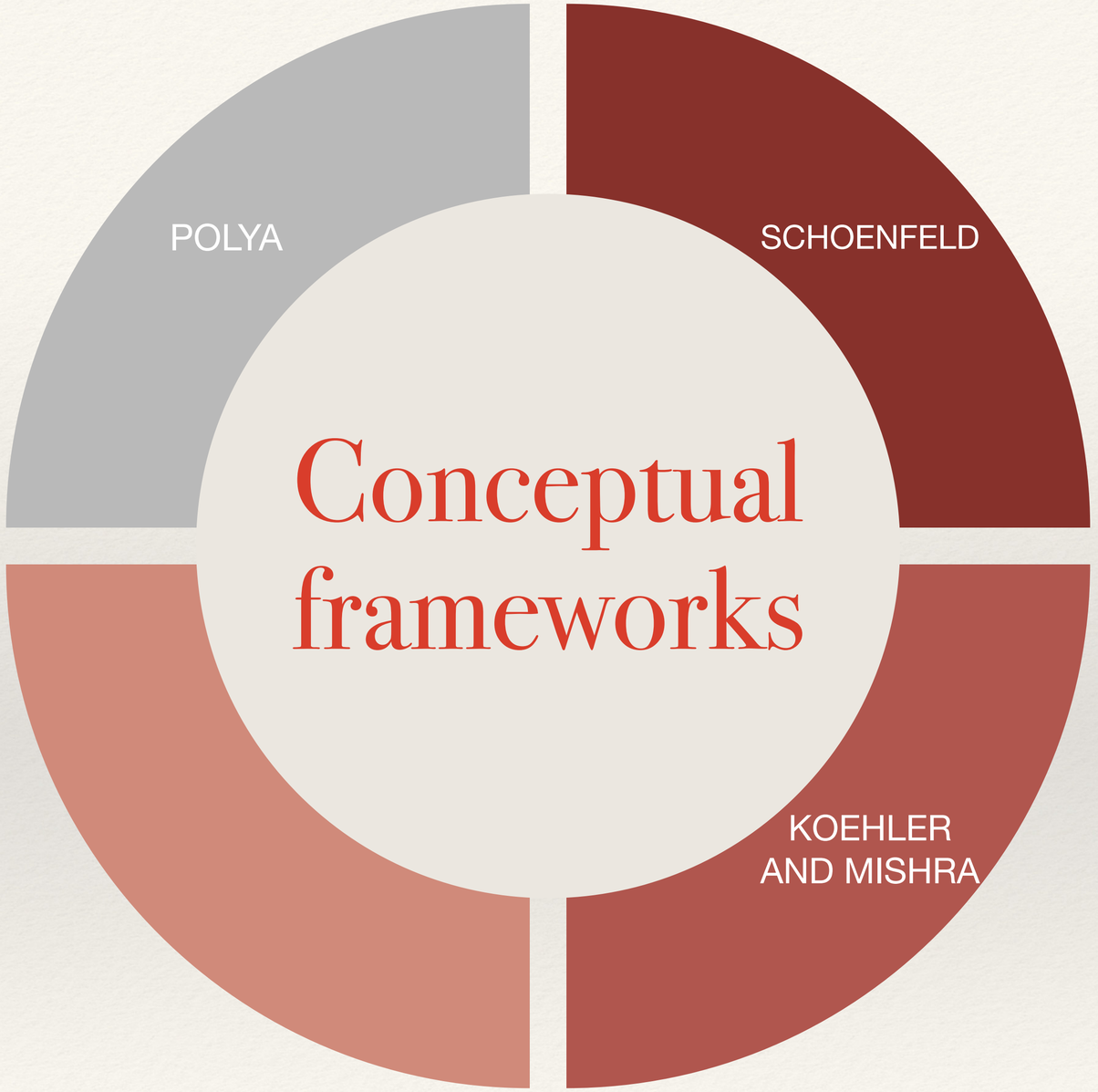


Polya’s four intertwined problem-solving phases: Understanding the problem, devising a solution plan, carrying out the plan, and looking back. The importance of heuristics methods (Based on his own experience, introspective approach, 1945)



Conceptual frameworks

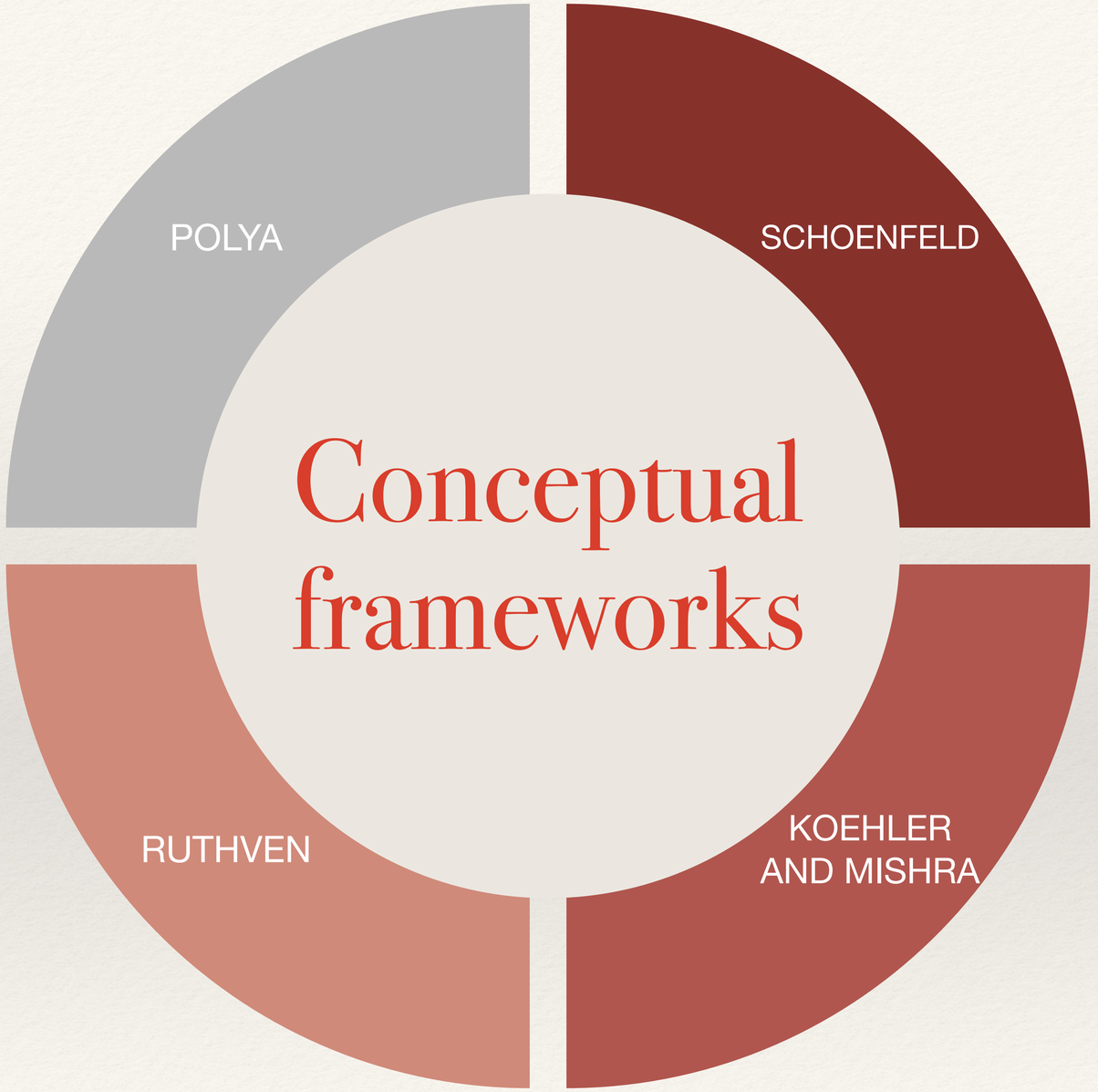
Schoenfeld, 1985: Four dimensions that shape the students' problem solving performances: Resources or knowledge base to face problems, the use of heuristics to understand problem statements; the use of self-monitoring and control strategies to make decisions; and the students' conception and beliefs about mathematics and problem solving



Conceptual frameworks

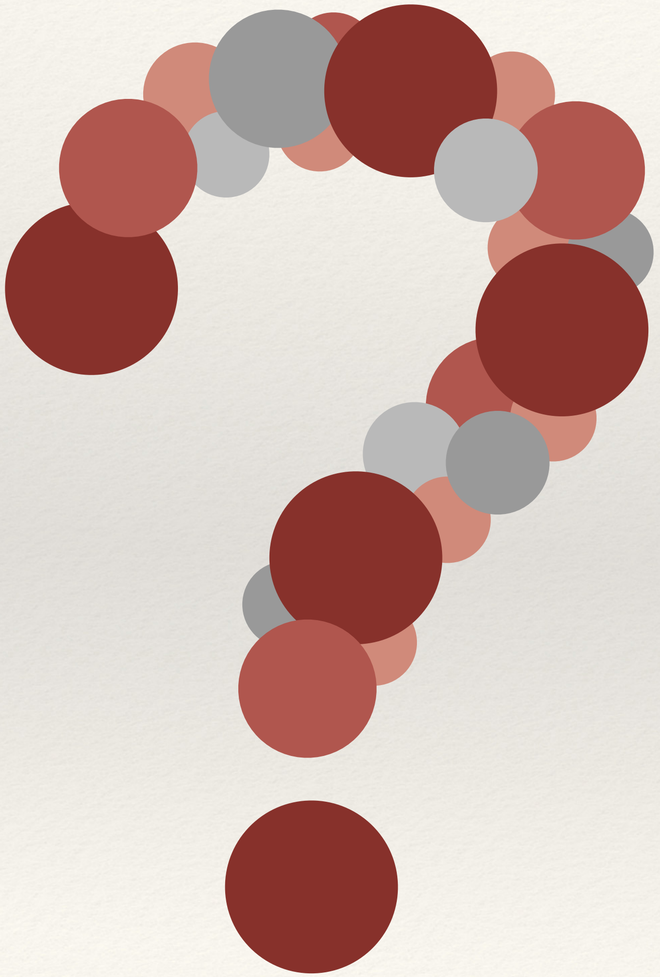
Koehler and Mishra (2011) made explicit the importance of integrating content knowledge with pedagogical and technology affordances to frame the use of technology in teaching/learning environments

Conceptual frameworks



Ruthven (2022) proposed three dimensions to support the integration of digital technologies in school mathematical practices. The ergonomic (interaction between humans and digital tools), epistemological (disciplinary and didactical knowledge to use the tools), and the existencial dimensions (conception of self and subject that shape the use of the tools).

Essential components to frame problem-solving instruction

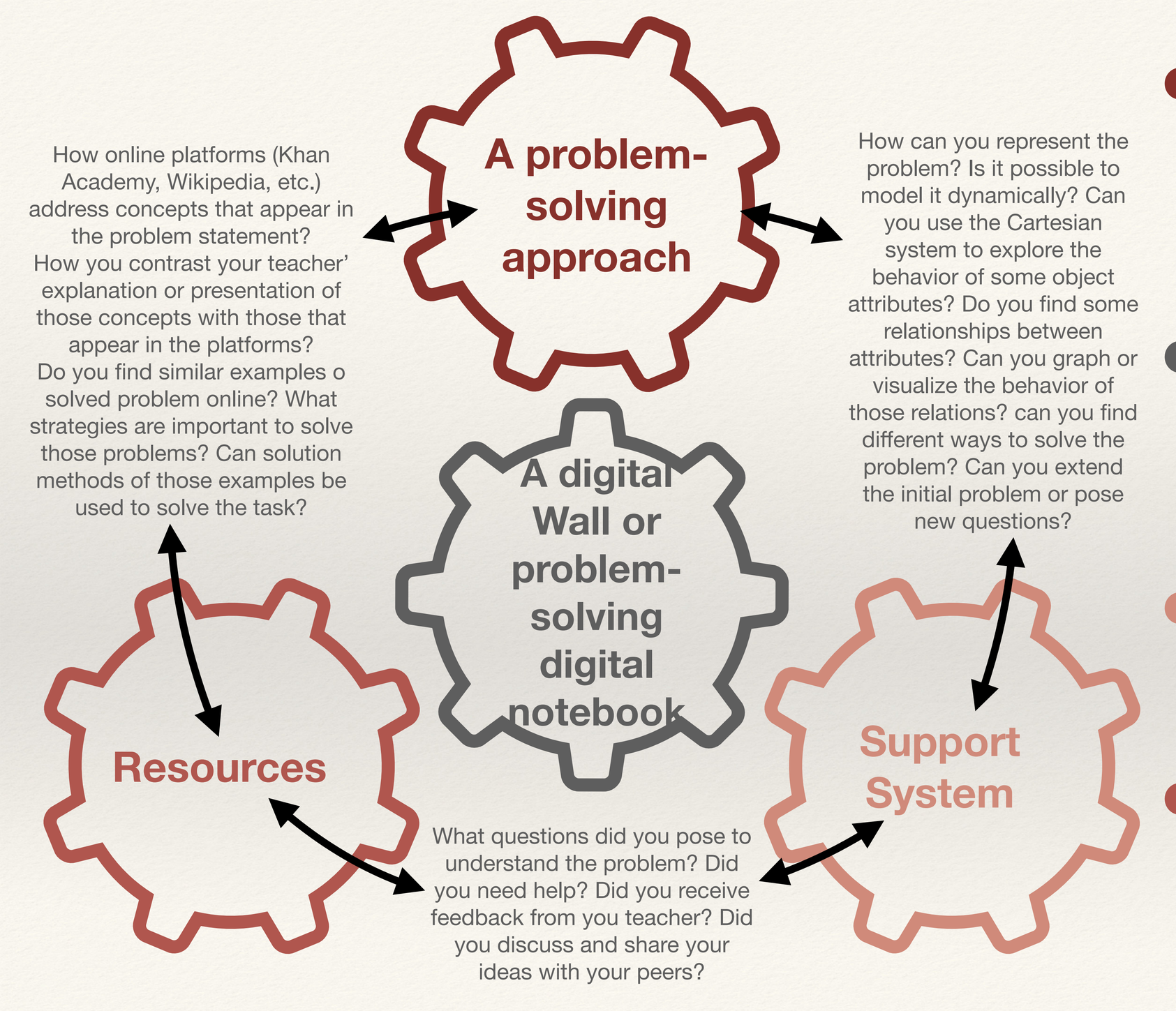


- An inquisitive or inquiry approach to delve into concepts and to work on mathematical problems. Mathematics as a set of dilemmas to elucidate and solve

- Tasks, problems or mathematical situations, embedded in different contexts, are the vehicle to engage students in mathematical practices and posing and discussion questions are key activities for students to learn concepts and to solve problems

- Looking for multiple or different ways or methods to represent and solve mathematical problems is important for learners to contrast concepts and strategies associated with each solution approach

- Learning mathematics and solving problems involve a continuous process in which students openly discuss and refine their ideas within a community that values and foster individual and collective participation and contributions



A problem-solving approach

- An inquiry or questioning method
- Different types of tasks and contexts (cognitive demands)
- Sense making activities and habits of mind
- Multiple ways to approach the task
- Learning as a continuous refinement process

A digital Wall or problem-solving digital notebook

- Short videos
- Students' notes
- Students' questions and concerns
- Problem solutions
- Posed problems
- Students' self-assessment

Support system

- Teacher and experts' feedback
- Peers' assessment
- Synchronous tools (chats) and discussion forums

Resources

- Online platforms (Khan Academy, Wikipedia)
- Communication Apps (Zoom, Teams, etc.)
- DGS (GeoGebra)
- Discussion Forums

A task: What mathematical questions could you formulate?

A tank or container of water is filled with one tap in four hrs and another tap fills the same tank in 6 h, how much time is needed to fill the same tank when both taps are open at the same time?

The use of a Cartesian system, slopes

A dynamic model

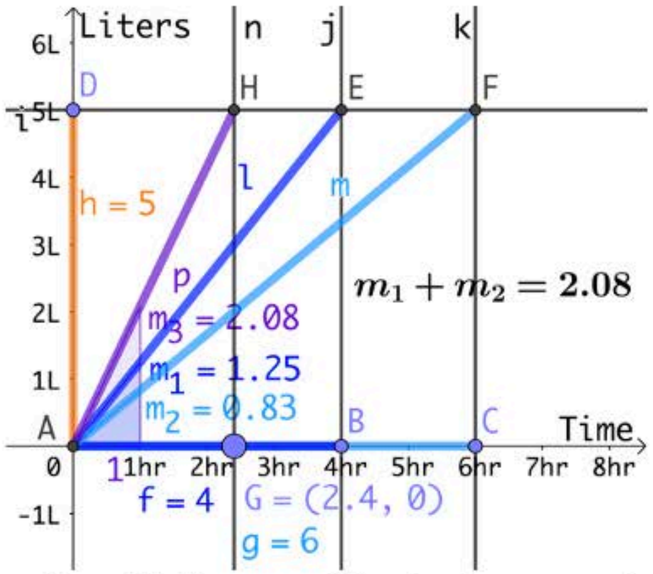


Figure 5a: Using a Cartesian system to represent key information of the problem

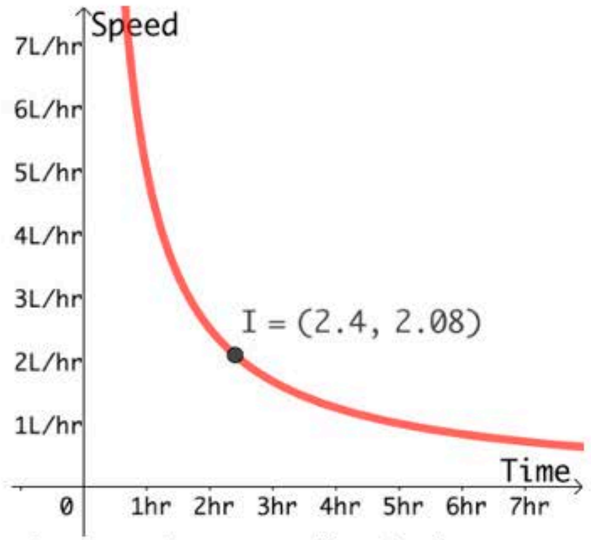
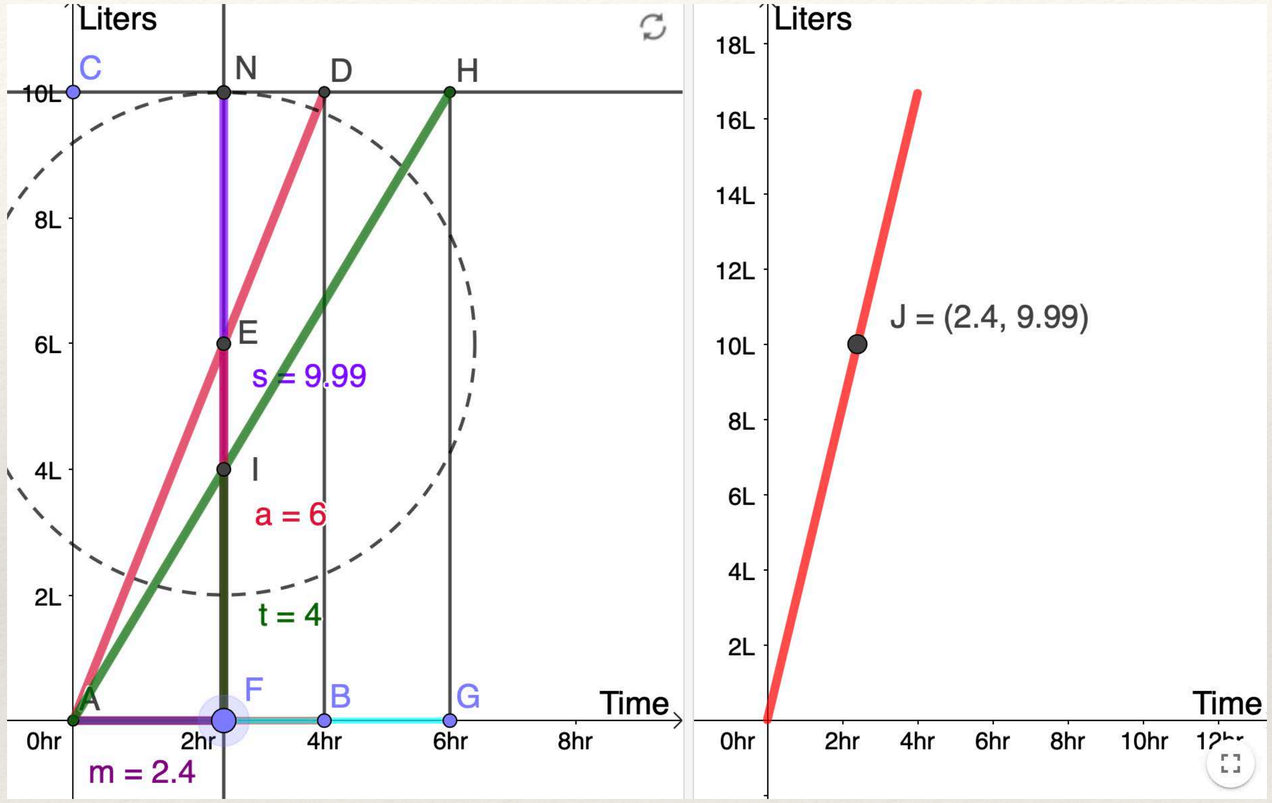


Figure 5b: Exploring the behaviour of the slopes of a family of segments AH that appears when point G is moved along x-axis

Dynamic model

The use of a Cartesian system, volume



Dynamic model

A problem-solving approach

An algebraic approach
A dynamic model
Focusing on variation of two sides of octagon and the position of E
Connecting the length of DE and ratio of EL & EF

How online platforms (Khan Academy, Wikipedia, etc.) address concepts that appear in the problem statement?
How you contrast your teacher' explanation or presentation of those concepts with those that appear in the platforms?
Do you find similar examples o solved problem online? What strategies are important to solve those problems? Can solution methods of those examples be used to solve the task?

How can you represent the problem? Is it possible to model it dynamically? Can you use the Cartesian system to explore the behavior of some object attributes? Do you find some relationships between attributes? Can you graph or visualize the behavior of those relations? can you find different ways to solve the problem? Can you extend the initial problem or pose new questions?

A digital Wall

Resources

Problematizing the use of Wiki contents
Analysis and critic of Khan Academy videos and teacher's lectures

What questions did you pose to understand the problem? Did you need help? Did you receive feedback from you teacher? Did you discuss and share your ideas with your peers?

Support System

Teacher's short online lectures
Teacher' feedback via chats and email

A problem-solving approach

- What elements and data are important in the picture?
- How long does it take to fill the container with tap 1 or 2?
- What about filling the same container with opening the two taps?
- How can you represent the relevant information in a Cartesian system?
- Can you associated the filling rate of each tap with the slope of a line?

A digital Wall or problem-solving digital notebook

- Short videos: word problems, slope, linear model
- Students' questions and concerns
- Problem solutions
- Posed problems
- Students' self-assessment

Support

- Technical support
- Teacher and experts' feedback
- Peers' assessment
- Synchronous tools (chats) and discussion forums

Resources

- Online platforms (Khan Academy, Wikipedia)
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The construction of a dynamic model: How to represent the given perimeter and diagonal geometrically?

An ellipse approach

Representing and exploring phenomena that involve change or variation

Can you draw a rectangle if you know its perimeter and its diagonal?

A dynamic model and a variation task

Remarks

Questions they pose to understand concepts and problem statements

Online resources and platforms they consult to contextualize problems and review and extend their understanding of involved concepts

Different ways to solve a mathematical problem. The type of problems for students to work include problems like those discussed during the class, those that can be solved by the same methods but differ from those solved in instruction and new problems that were not addressed in class sessions.

Identification and analysis of oncepts and strategies used to solve the problem

Remarks

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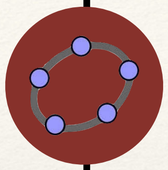
Concepts and strategies used to solve the problem

Identification of other problems that can be solved with the methods that were used to solve the initial problem

Remarks

Digital technologies and online resources used to solve the problem

Dynamic models used to solve the problem and strategies used to identify and explore mathematical relations (dragging objects, measuring object attributes, tracing loci, using sliders, etc.)



Formulation of new related problems including possible extensions for the initial problem

Discussion of solutions of some new problems

Short recorded video presentation of their work and problem solutions

Remarks

Reflection on how their problem solutions relied on other peers' ideas and the extent to which their own work influenced and shaped the group work

Thank you