

What and how to teach mathematics for the future?

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Imagine the future in 2030 or 2040

Challenges
Solutions

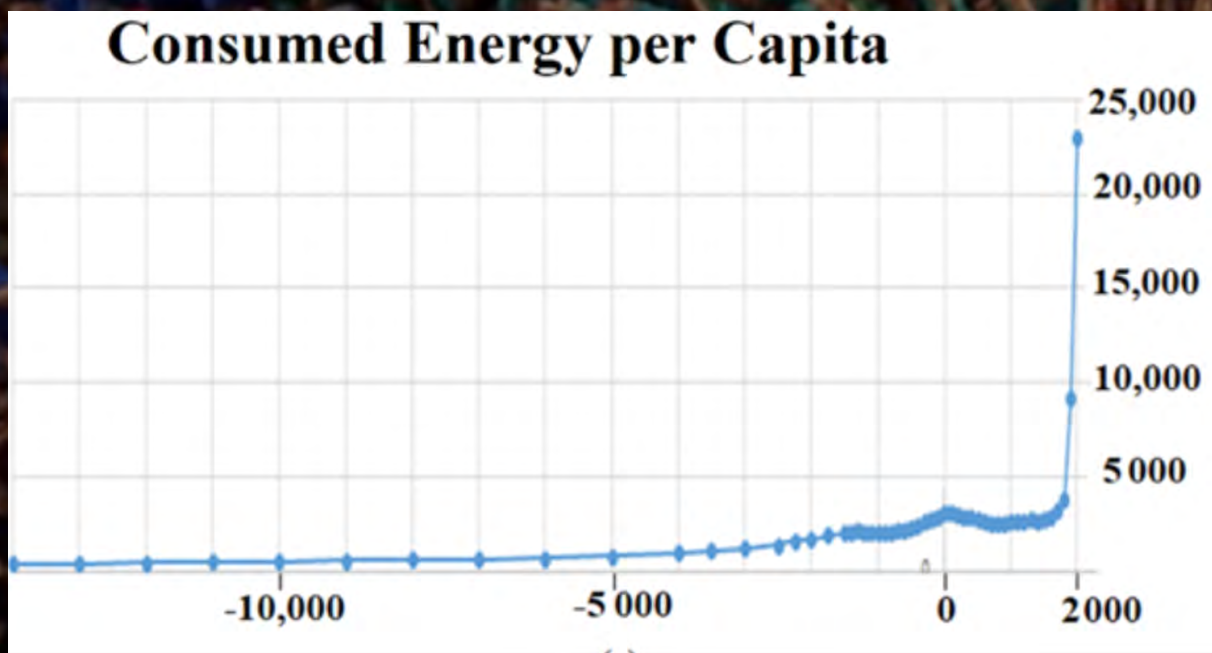
We are experiencing a great acceleration in social change and a true explosion of knowledge of the natural and social world.

It is natural then to ask whether we have to update school mathematics content.

What we now teach was mainly developed in agricultural and previous commercial societies.

Now we are globally interconnected. Every day, an increasing number of autonomous agents interact with us. Moreover, they are starting to decide for us.

Climate change



- Morris, I.
The Measure of Civilizations



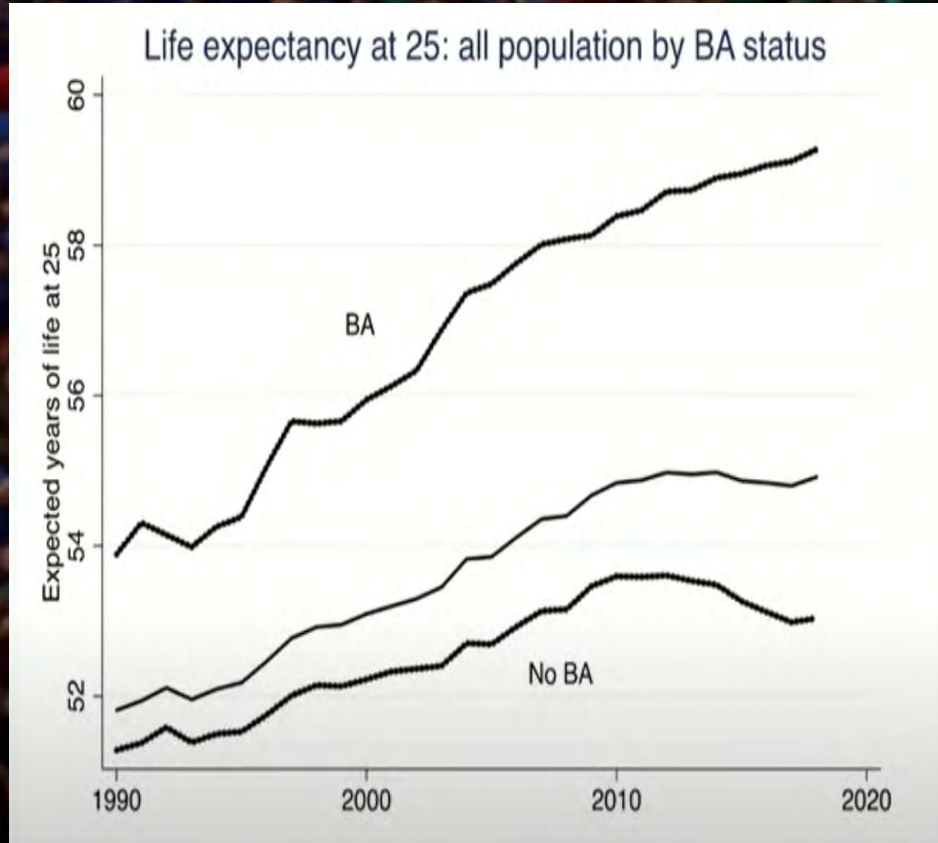
Pandemics



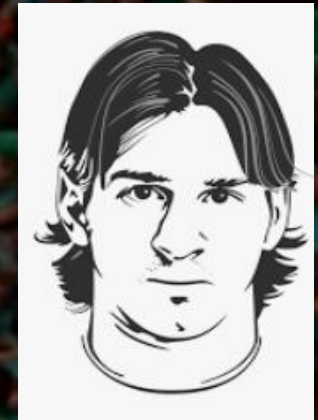
Inequality

The defining challenge of our time

- President Obama

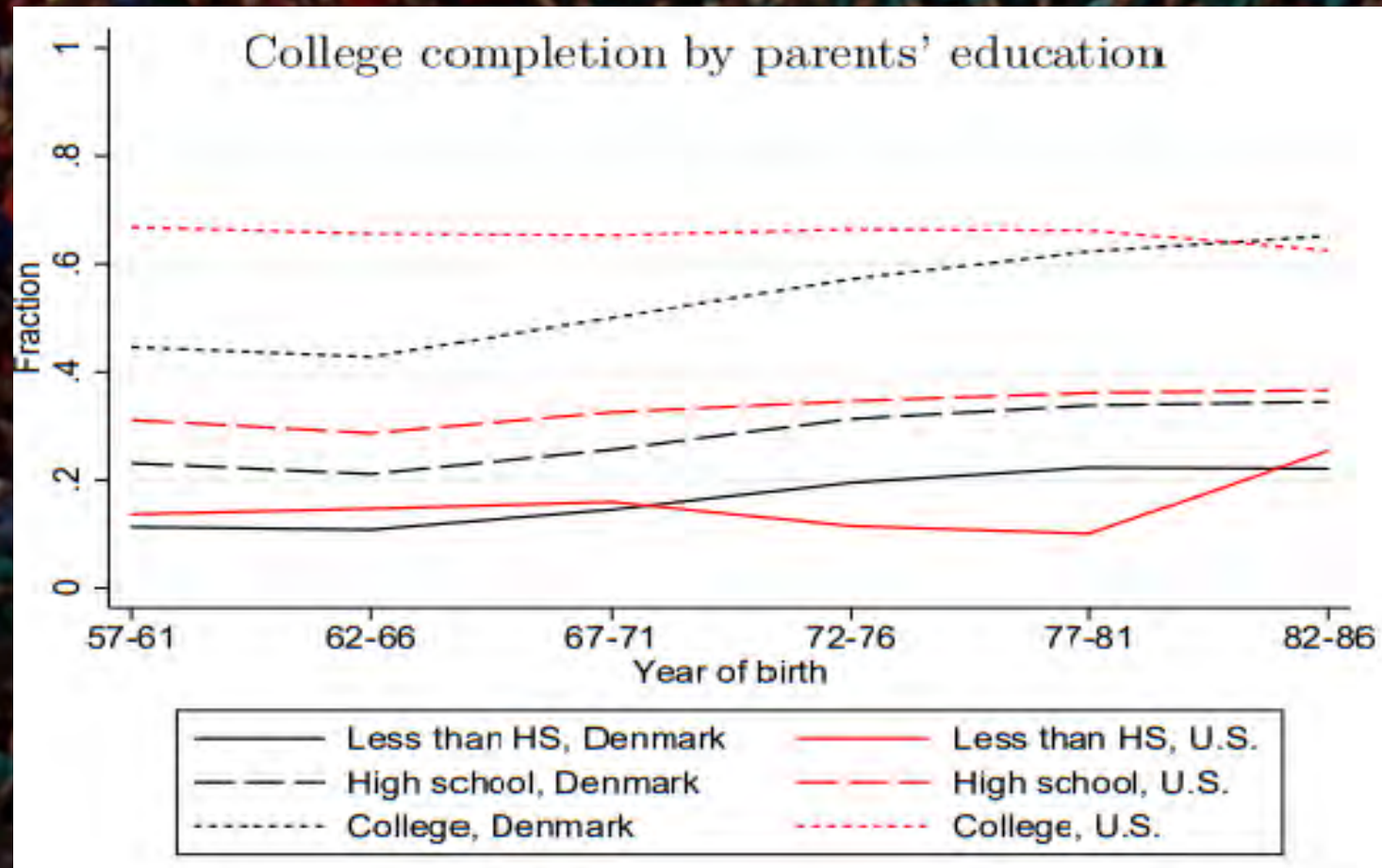


- Nobel Laureate Angus Deaton
Inequality and the future of capitalism



– Frank & Cook
The Winner-Take-All Economy

Inequality



- Nobel Laureate J. Heckman & R. Landersø
Lessons from Denmark about inequality and social mobility

AI

Unlike the previous three challenges, this challenge is a major turning point in the history of civilization.

Autonomy, free will, and the nature of the human condition are at stake.

Given these implications, we will primarily focus on this challenge.

I postulate 3 conjectures

First, the main challenge is trust. What agents should we trust? Here we have an evolutionary mismatch. Our brains evolved to trust a small group of people, not billions. Much less to trust in smart apps. We need to provide citizens with tools to solve this challenge.

Super Smart Society

Your smartphone talks to you and makes suggestions

Your smartphone takes the initiative and decides for you

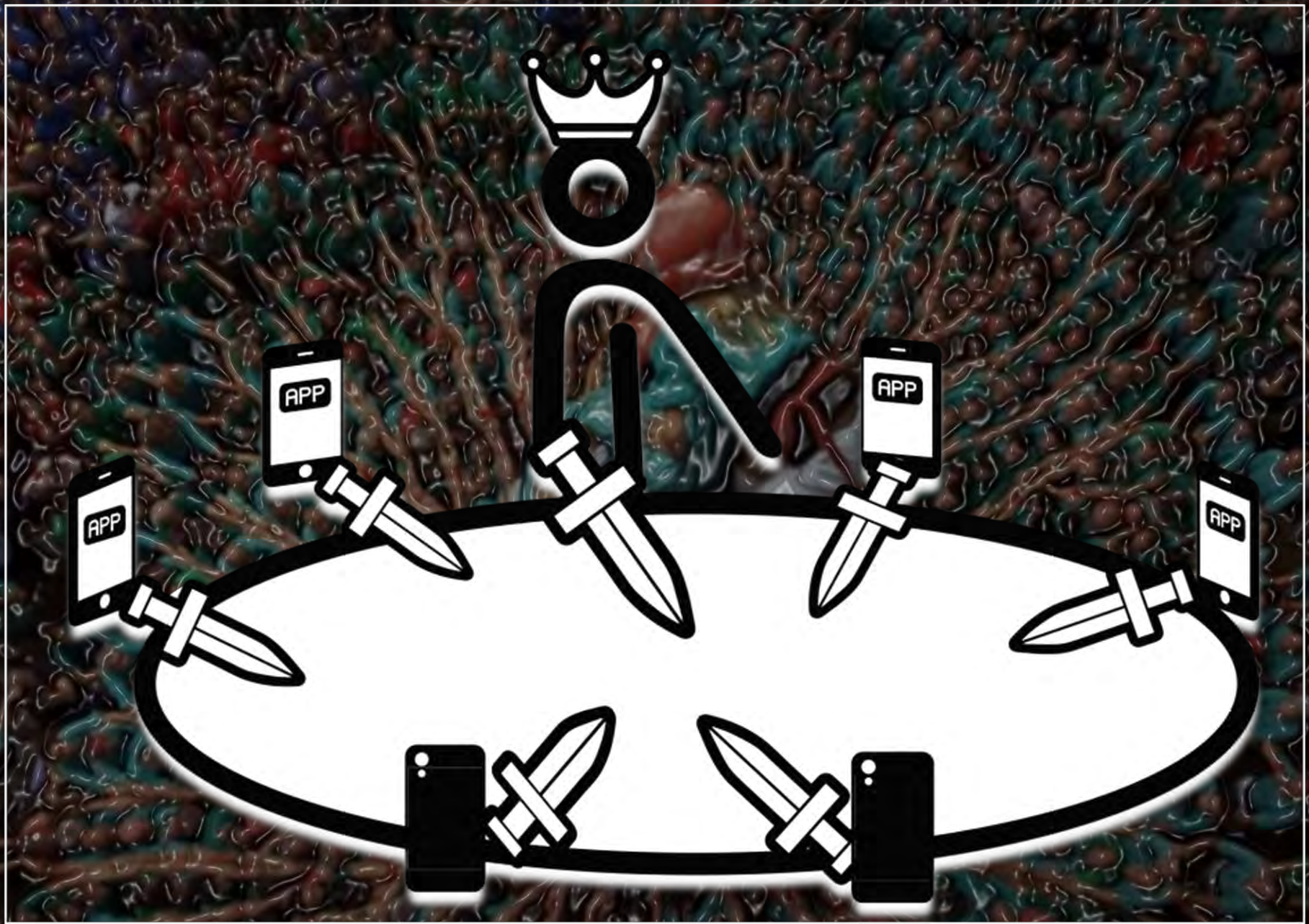
Your smartphone learns autonomously

A second App on your smartphone also takes the initiative but has another opinion

Both Apps take the extra initiative to talk to each other and discuss their arguments.

You have 10 different Apps with different personalities

Two gangs of Apps are formed, accusing each other of having unfair biases and preconceptions, corruption, and treason



Trust





An evolutionary mismatch



150

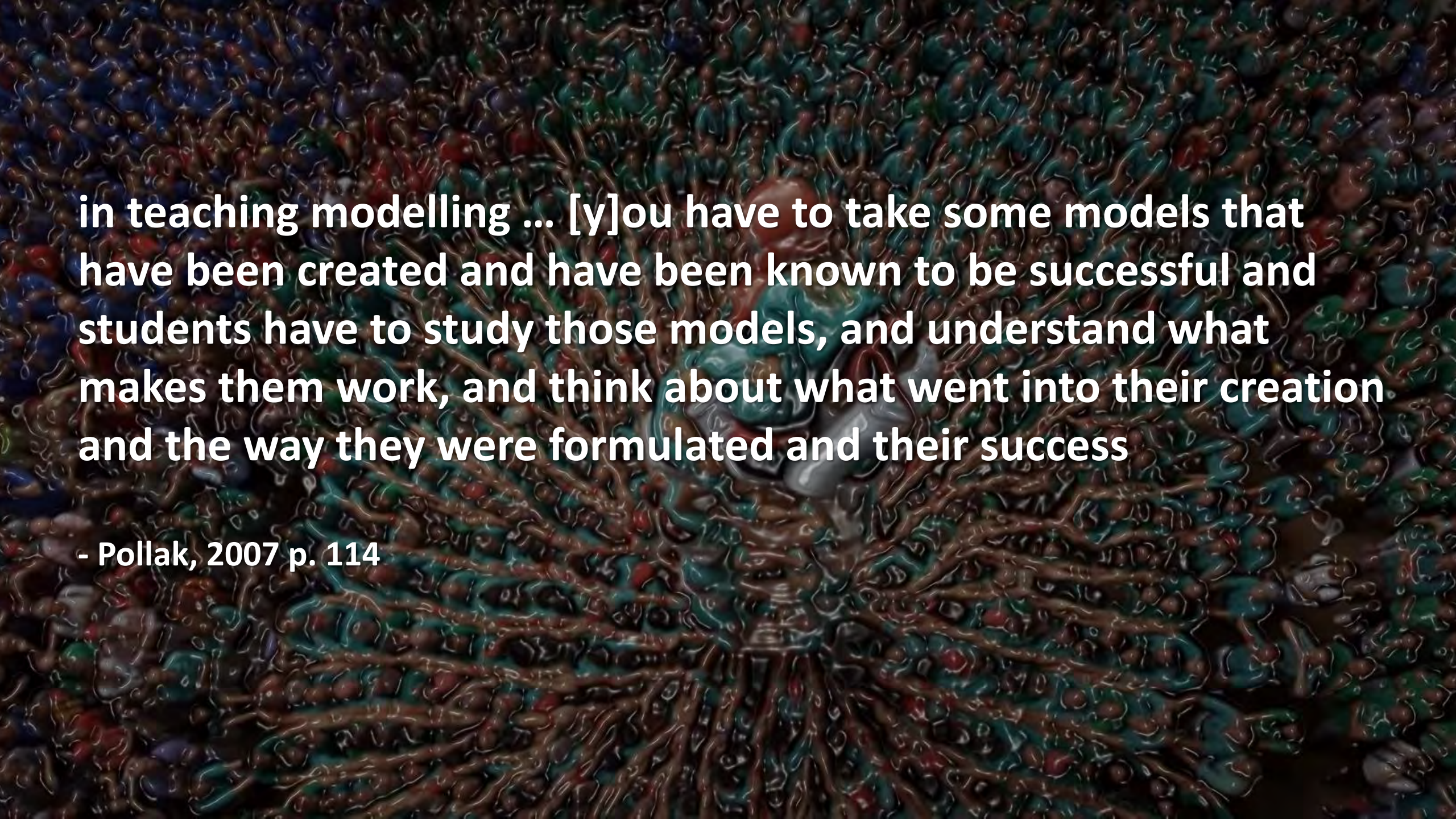
Dunbar number

An evolutionary mismatch

I postulate 3 conjectures

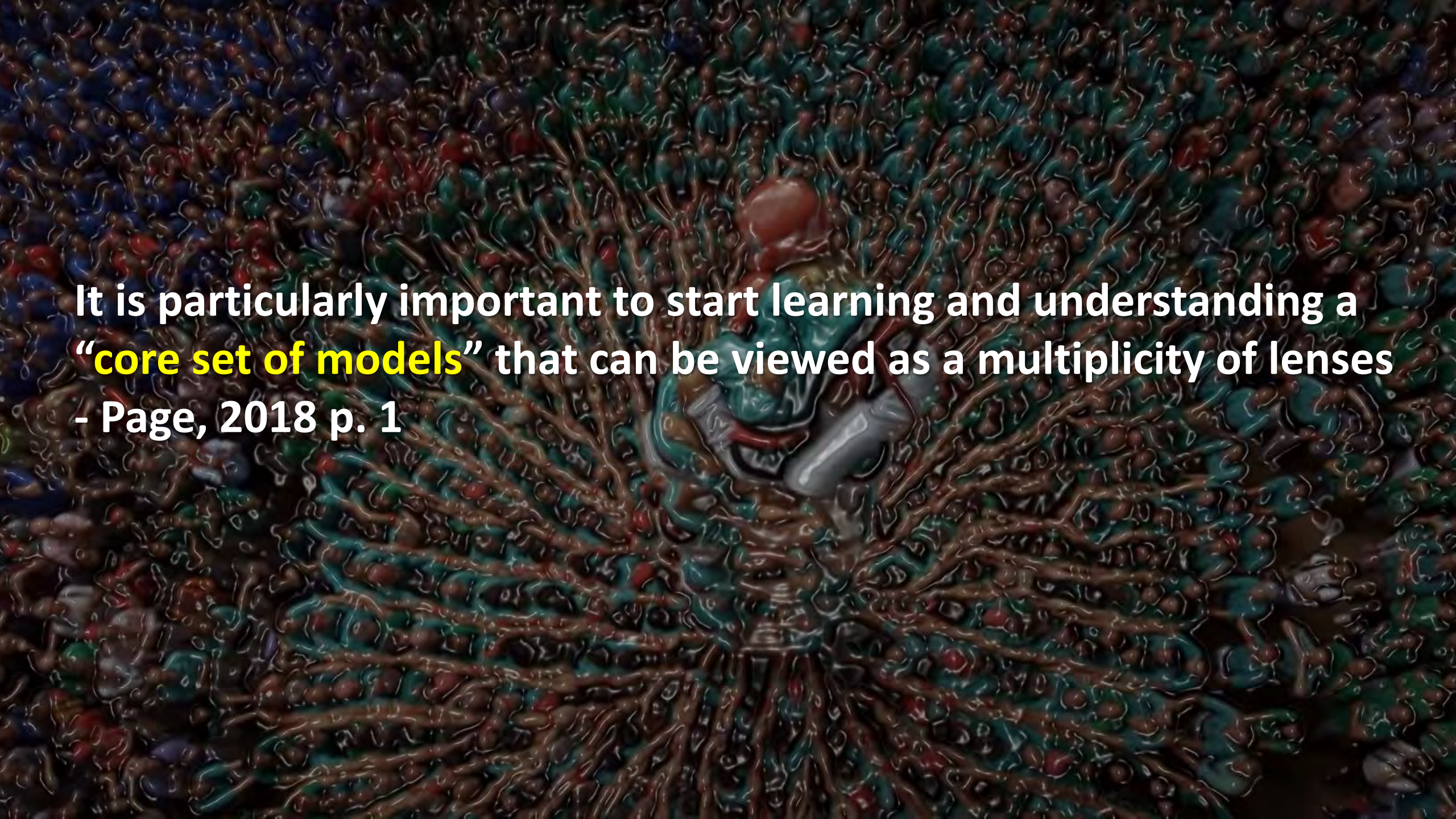
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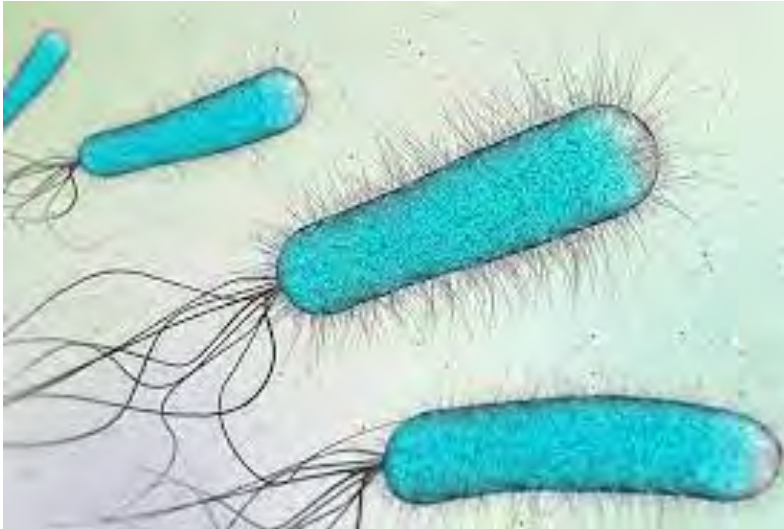
in teaching modelling ... [y]ou have to take some models that have been created and have been known to be successful and students have to study those models, and understand what makes them work, and think about what went into their creation and the way they were formulated and their success

- Pollak, 2007 p. 114

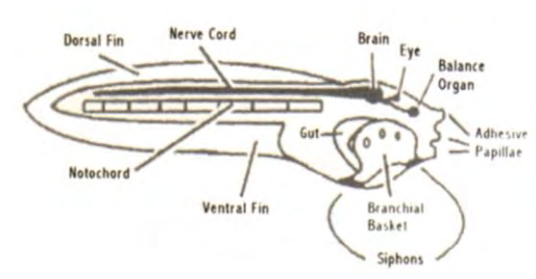
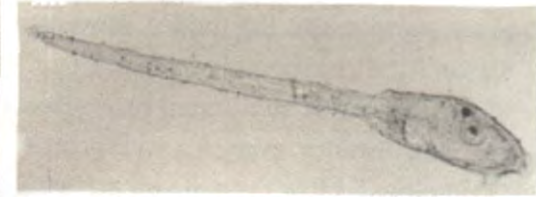
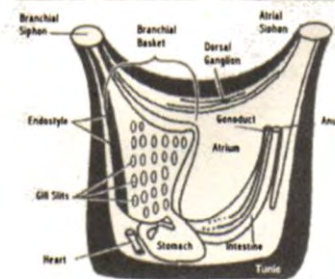
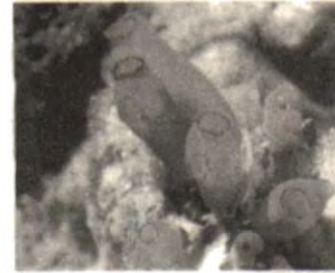


It is particularly important to start learning and understanding a
“**core set of models**” that can be viewed as a multiplicity of lenses
- Page, 2018 p. 1

Agent Navigation



Bacterial navigation



Sea squirt

Use - Select - Adjust - Build

Mark the path that the bacteria must follow, which always moves to the neighboring cell where the higher number is (the number represents the amount of nutrients), until it no longer finds a higher number.

Mark the path of the bacteria that begins above where the two is located.

Mark the path of the bacteria that begins above where the five is located.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 2 | 1 | 3 | 5 | 6 | 3 | 3 | 4 | 3 | 0 |
| 10 | 2 | 2 | 2 | 7 | 4 | 5 | 5 | 4 | 2 |
| 11 | 13 | 2 | 3 | 4 | 9 | 10 | 3 | 14 | 13 |
| 21 | 3 | 19 | 4 | 4 | 19 | 5 | 14 | 4 | 3 |
| 22 | 4 | 4 | 20 | 5 | 25 | 6 | 16 | 25 | 5 |
| 30 | 24 | 5 | 18 | 6 | 25 | 6 | 6 | 19 | 4 |
| 32 | 4 | 6 | 16 | 6 | 17 | 6 | 6 | 4 | 4 |
| 2 | 4 | 26 | 15 | 12 | 19 | 27 | 35 | 25 | 15 |
| 32 | 34 | 36 | 37 | 38 | 10 | 47 | 41 | 35 | 14 |
| 38 | 44 | 35 | 8 | 39 | 28 | 29 | 38 | 18 | 13 |
| 11 | 4 | 4 | 6 | 40 | 38 | 9 | 10 | 11 | 12 |
| 0 | 5 | 3 | 4 | 41 | 8 | 8 | 5 | 4 | 2 |

Mark the path of the bacteria that begins above where the zero is located.

In which model are there 2 bones?

Araya, R. (2021) [Enriching Elementary School Mathematical Learning with the Steepest Descent Algorithm](#). Mathematics 2021, 9, 1197

Araya, R. (2022) АЛГОРИТМ НАЙСКОРЕЙШЕГО СПУСКА ДЛЯ НАЧАЛЬНОЙ ШКОЛЫ. МАТЕМАТИКА февраль

Use - **Select** - Adjust - Build

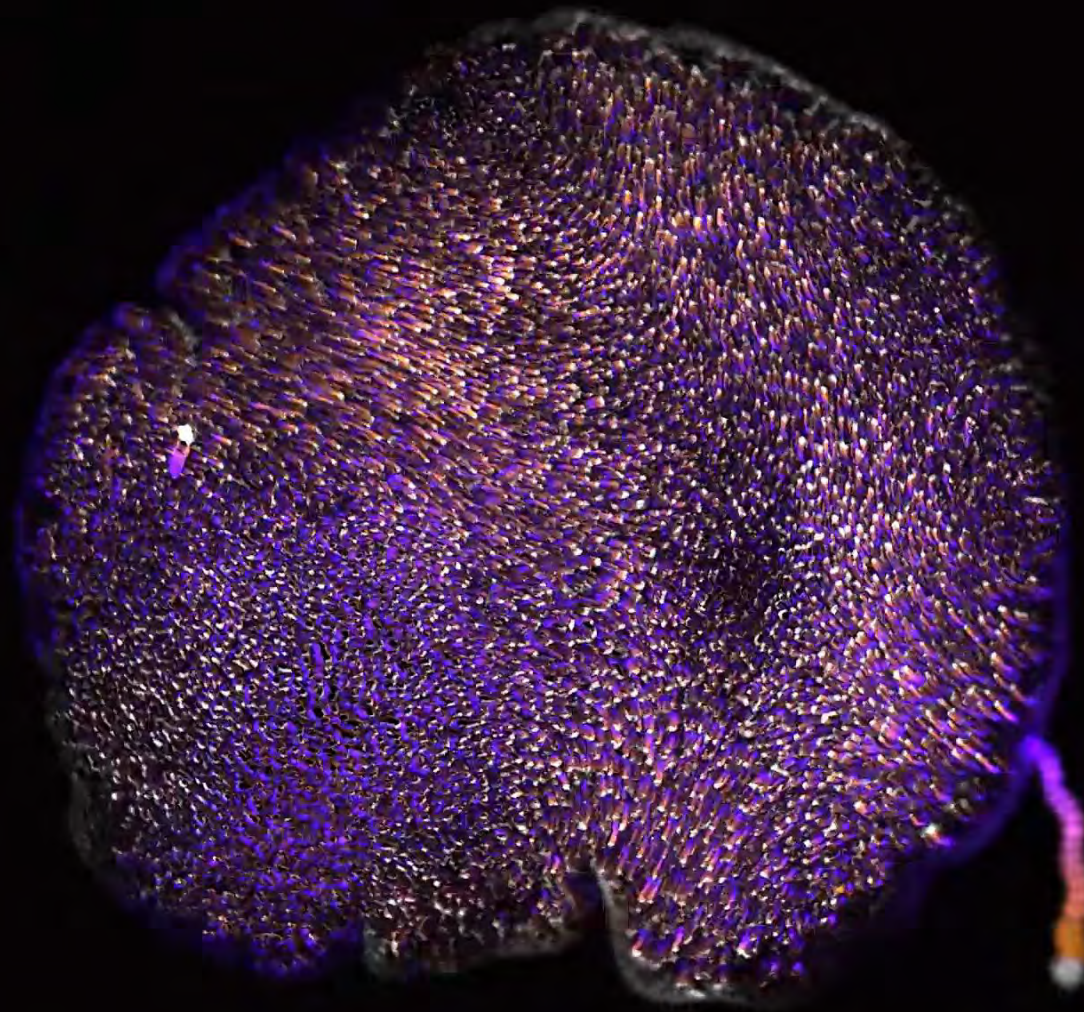
The hound always moves to the neighboring cell with the highest number (the number represents the intensity of the smell), until it no longer finds a neighboring cell with the highest number (where the bone is).

Connect the board that represents a courtyard where two bones are hidden in different places.

| | | | | | | | | | | | | | |
|----|----|----|----|----|----|---|----|----|----|----|----|----|----|
| 19 | 26 | 24 | 23 | 22 | 15 | 8 | 14 | 14 | 11 | 14 | 10 | 14 | 13 |
| 20 | 25 | 32 | 26 | 19 | 15 | 8 | 19 | 20 | 19 | 13 | 10 | 19 | 20 |
| 21 | 27 | 27 | 25 | 20 | 16 | 8 | 22 | 26 | 15 | 15 | 14 | 21 | 14 |
| 21 | 19 | 19 | 22 | 18 | 13 | 9 | 18 | 21 | 13 | 21 | 22 | 21 | 22 |
| 14 | 14 | 15 | 14 | 14 | 14 | 9 | 14 | 14 | 15 | 21 | 25 | 25 | 26 |
| 8 | 9 | 8 | 8 | 6 | 10 | 9 | 7 | 10 | 15 | 20 | 25 | 33 | 24 |
| 3 | 3 | 3 | 1 | 1 | 1 | 2 | 1 | 6 | 14 | 19 | 25 | 26 | 27 |

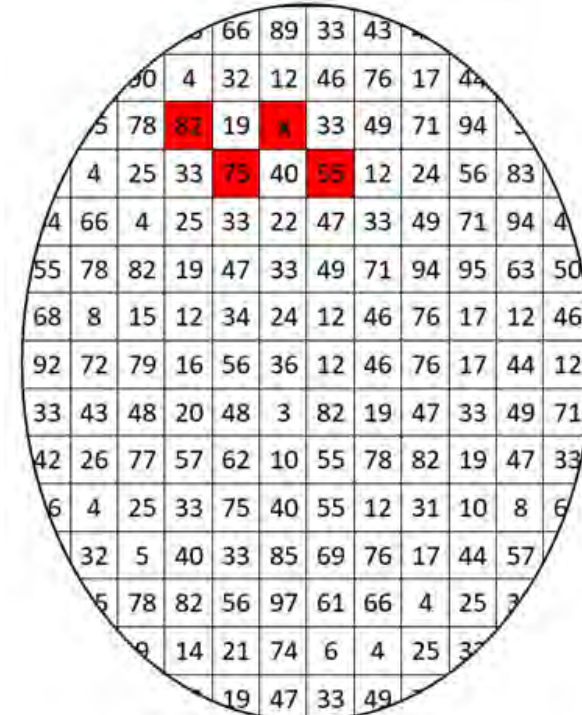
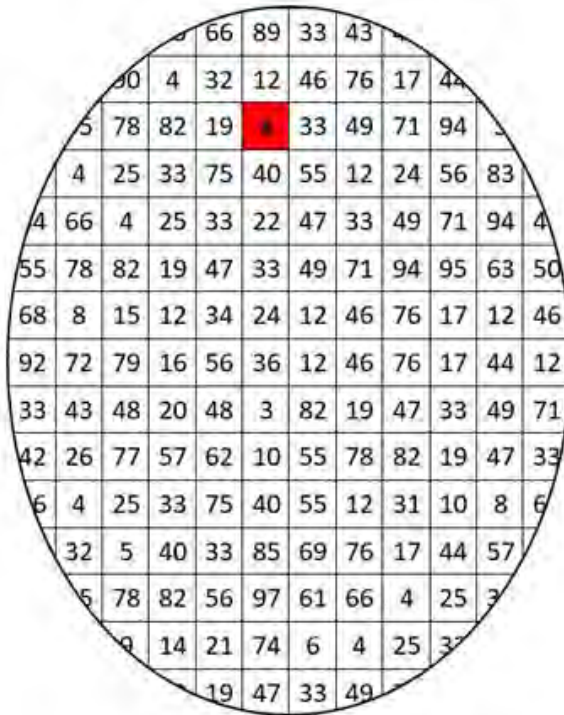
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Trichoplax don't have a nervous system

Gastrulation



Araya, R. (2022) Is it feasible to teach agent-based computational modeling to elementary and middle school students? Proceedings of the Singapore National Academy of Science. To appear.

Machine Learning

Steepest Descent Algorithm

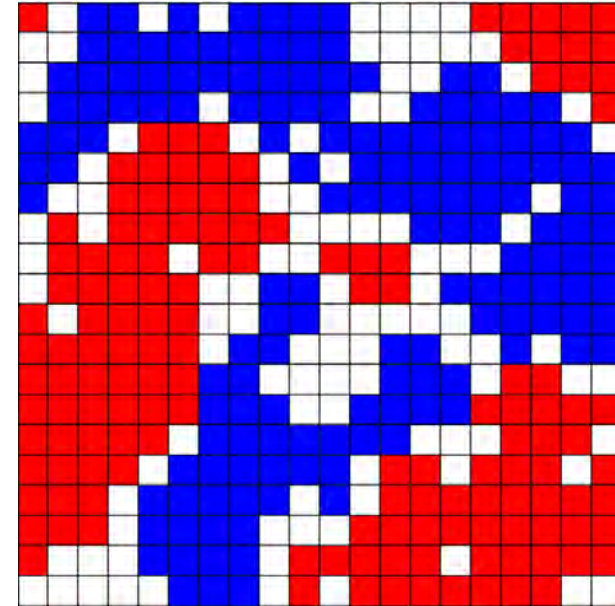
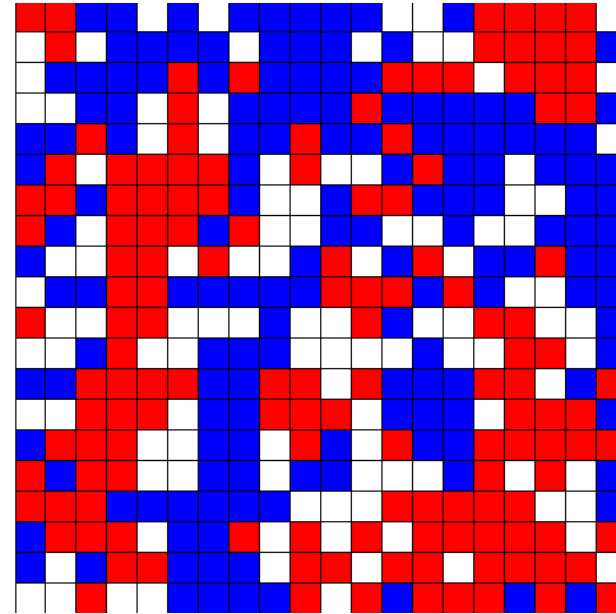
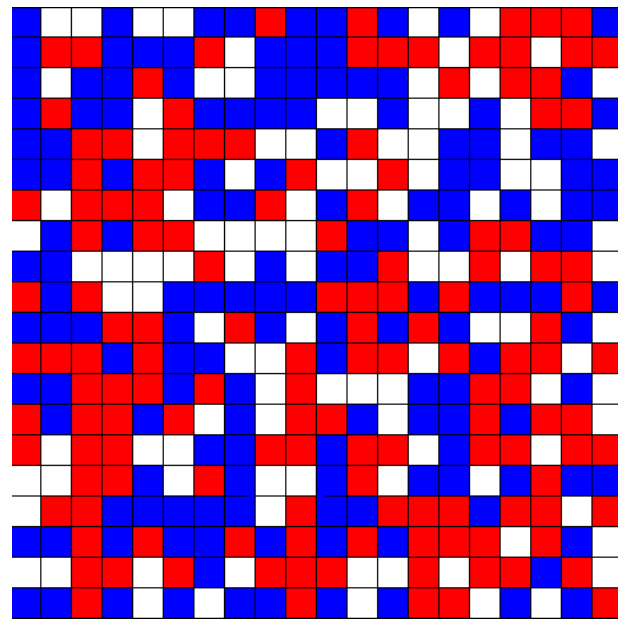
Gradient Descent Algorithm

Araya, R. (2021) [Enriching Elementary School Mathematical Learning with the Steepest Descent Algorithm](#). Mathematics 2021, 9, 1197

Araya, R. (2022) АЛГОРИТМ НАЙСКОРЕЙШЕГО СПУСКА ДЛЯ НАЧАЛЬНОЙ ШКОЛЫ. МАТЕМАТИКА февраль

Schelling segregation model

Use
Select
Adjust
Build



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Third, I propose that to find effective strategies on how to teach them, the solution lies in teachers' collaborative work. We need lesson studies 2.0. These are lesson studies enhanced with AI support. Recent advances in natural language processing, pattern recognition in videos, and machine learning provide us with a powerful microscope to analyze and rapidly adjust teaching practices to these new teaching challenges.



Lesson Study

Open Lesson in Japan



Open Lesson in Thailand



Lesson Study 2 APEC InMside: Algorithmic Thinking 8th Grade Daniela



Ver más tarde

Compartir

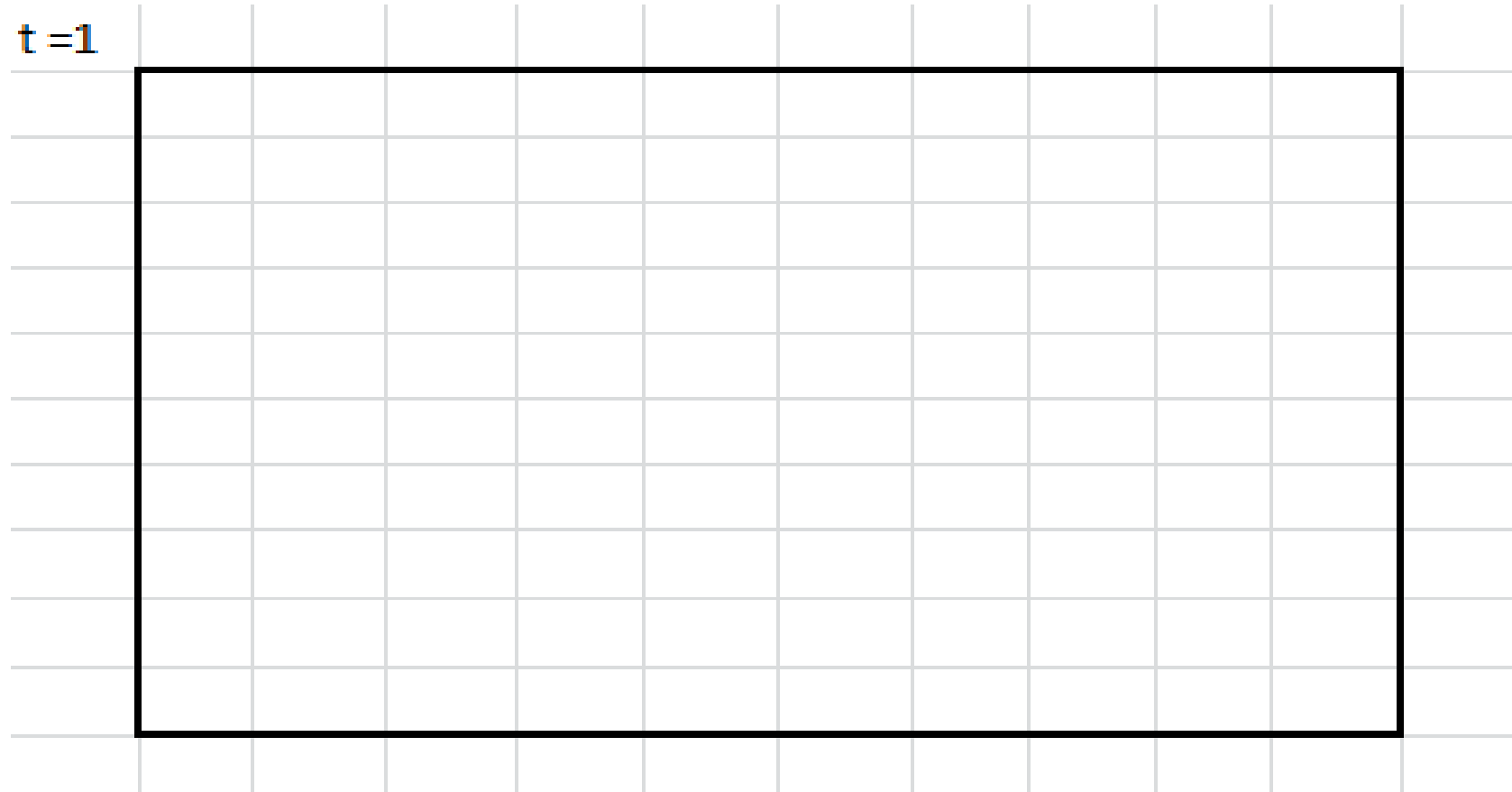


Computational modeling of forest fires

**Use
Select
Adjust
Build**

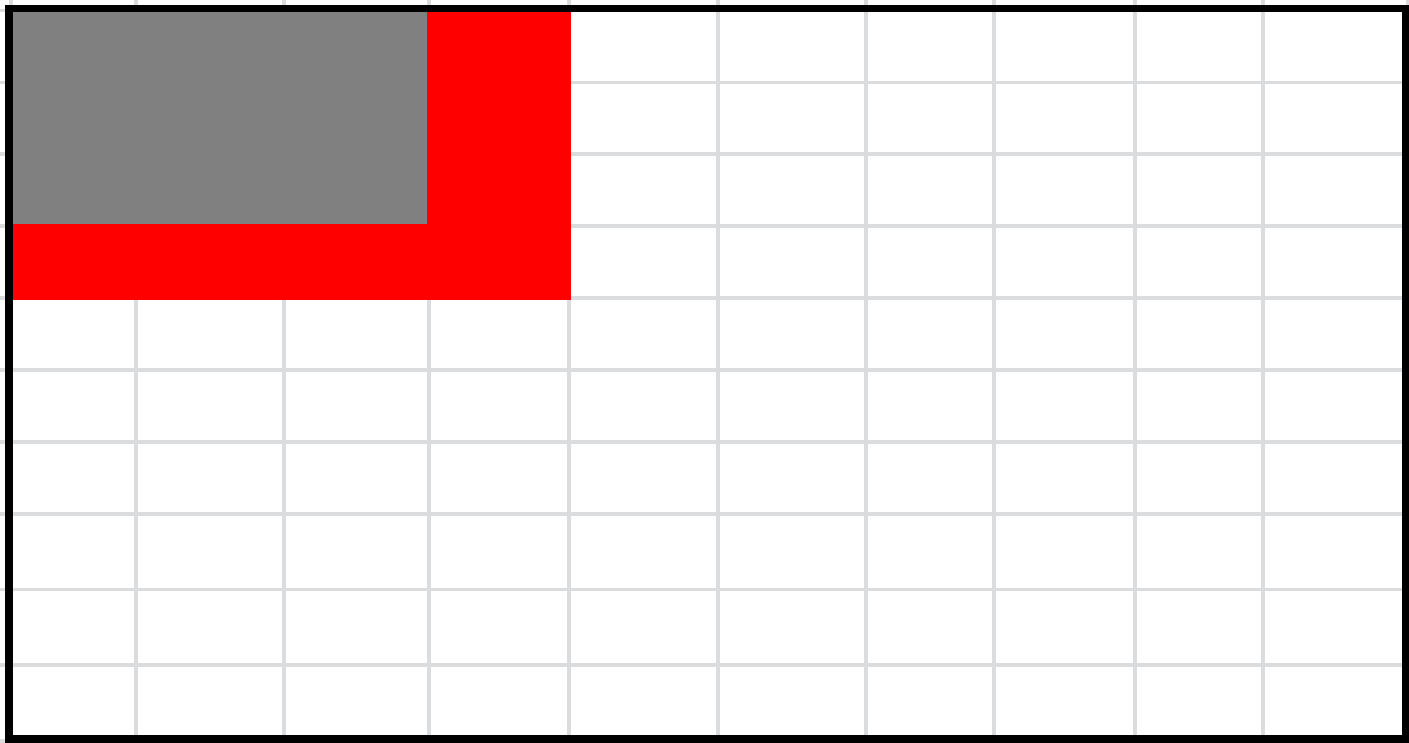




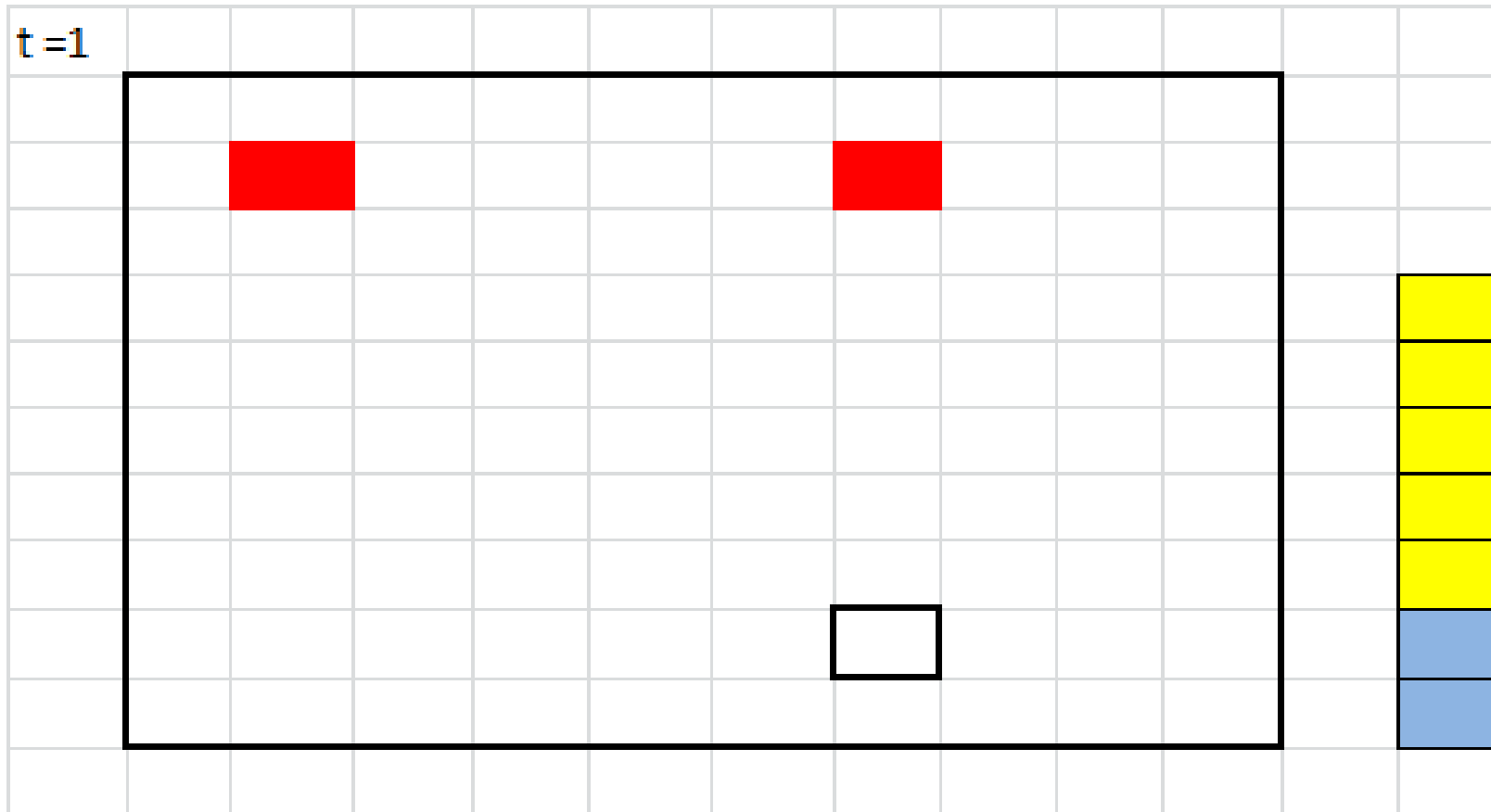


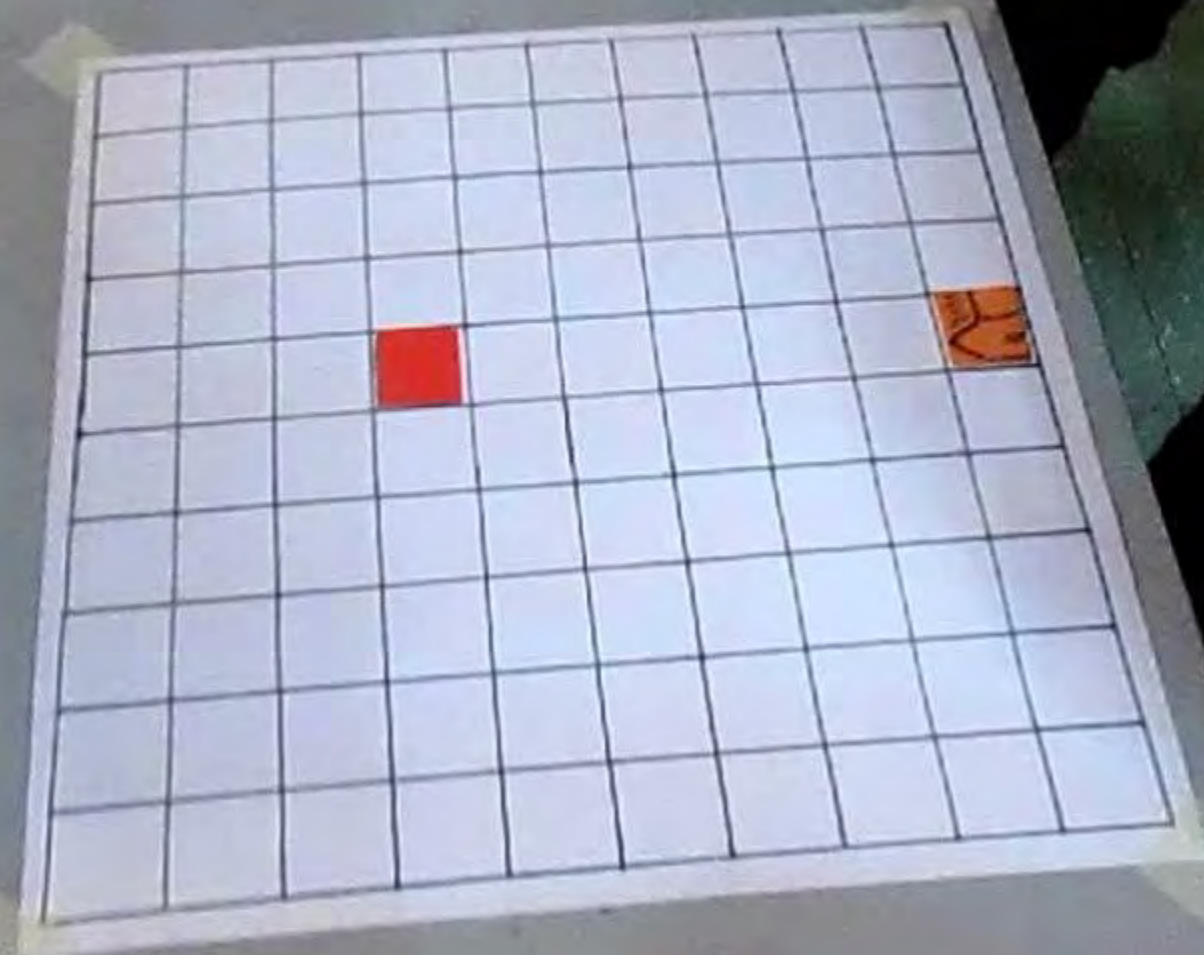




$$t=3$$


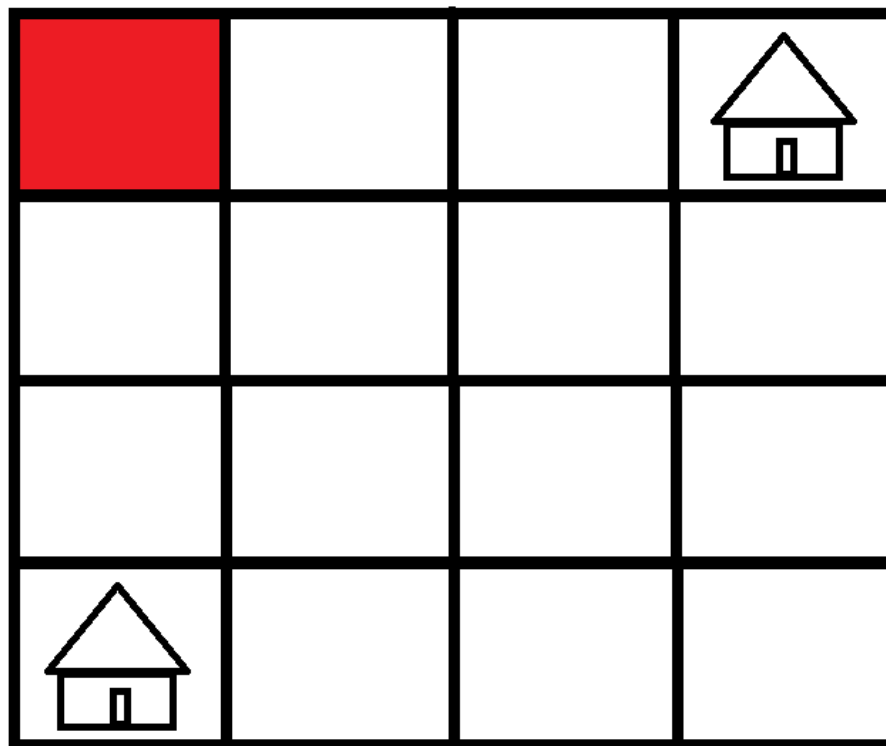
Can the house be saved?



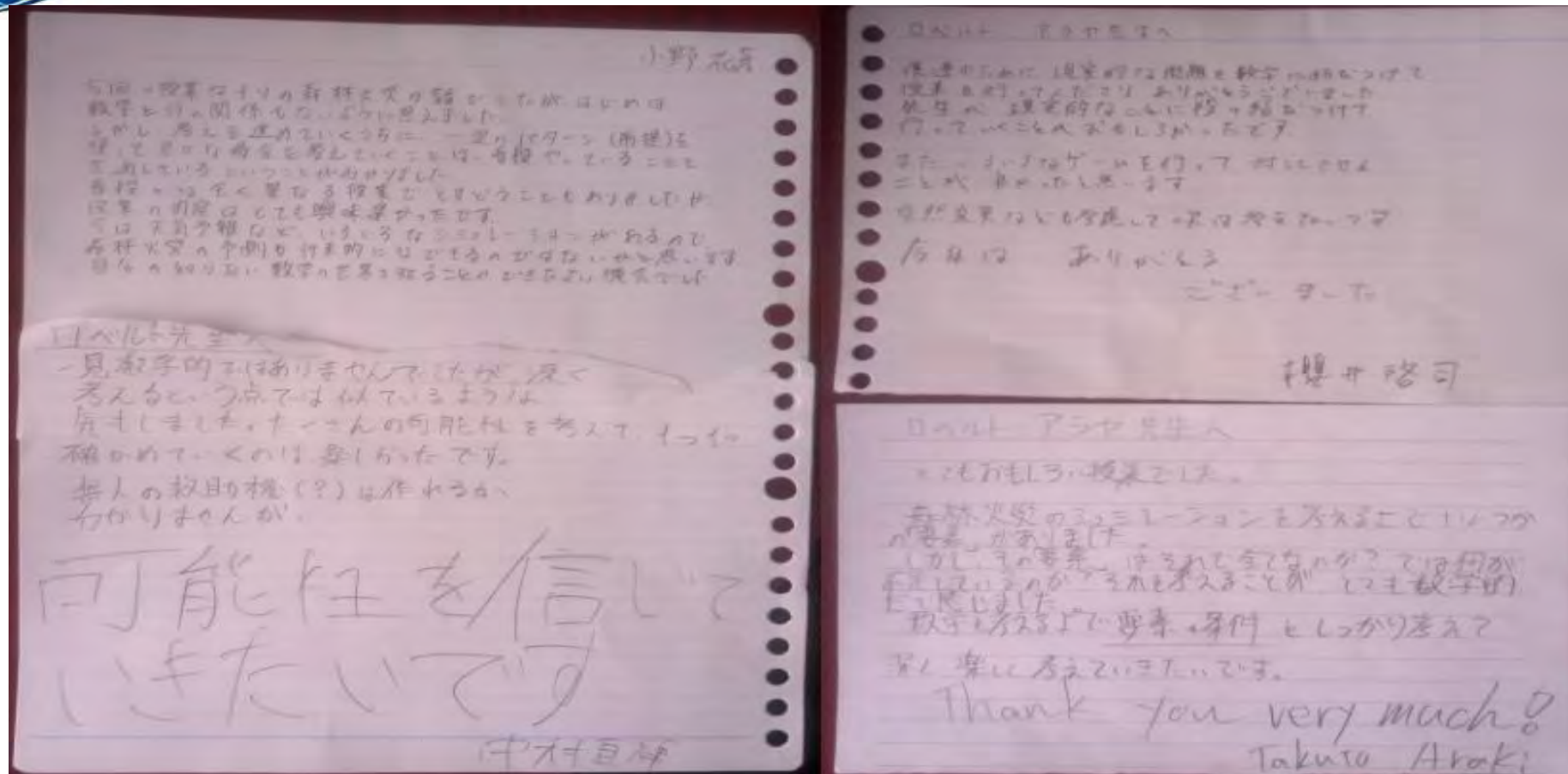




Can you save the 2 houses?







“I had a great time like never before. More than the typical style of classes in Japan”
- Takahashi

Dark matter

Lesson study

Find the time to meet face to face to observe the lessons of the other members of the group.

It works with small groups of teachers.

Although the open lessons take place with thousands of teachers actively observing the classes, they only take place a couple of times a year.

We need to build a much larger and more connected community. If the community is very small, we may experience the "*Tasmanian effect*"

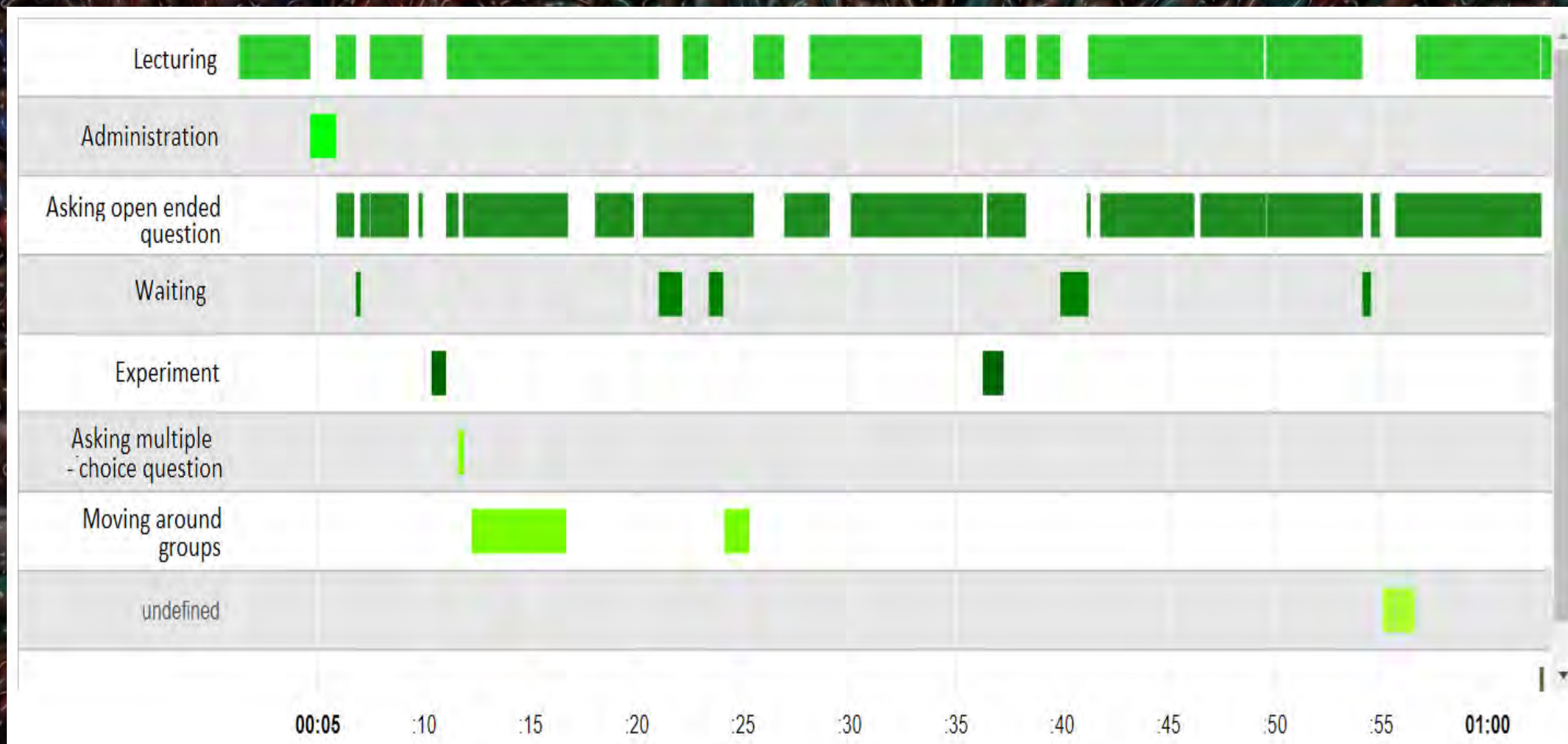
Lesson Study 2.0



COPUS protocol



COPUS Protocol



Lesson study with AI support

Speech

Acoustics

Transcripts

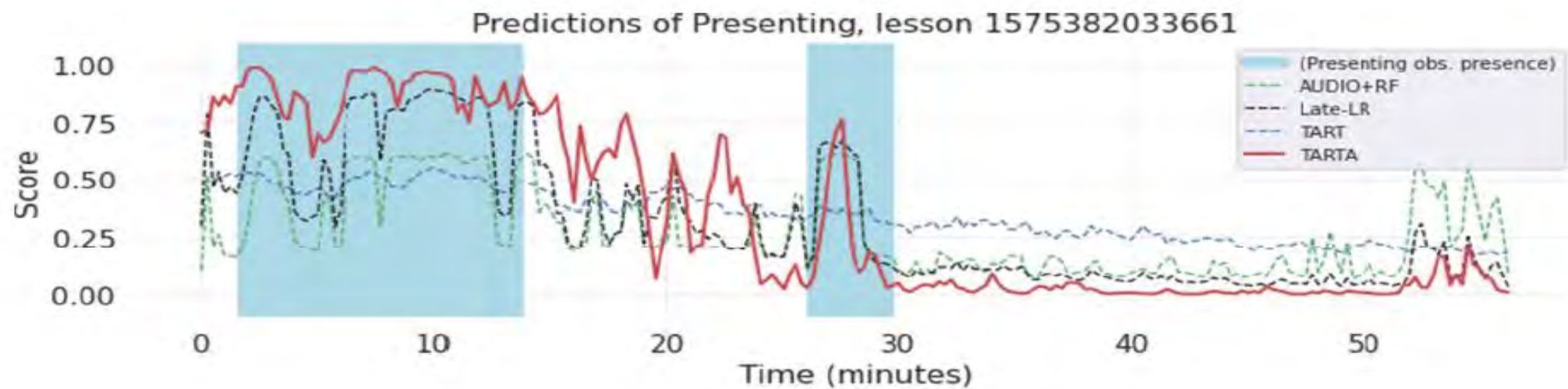
Non-verbal

Gestures

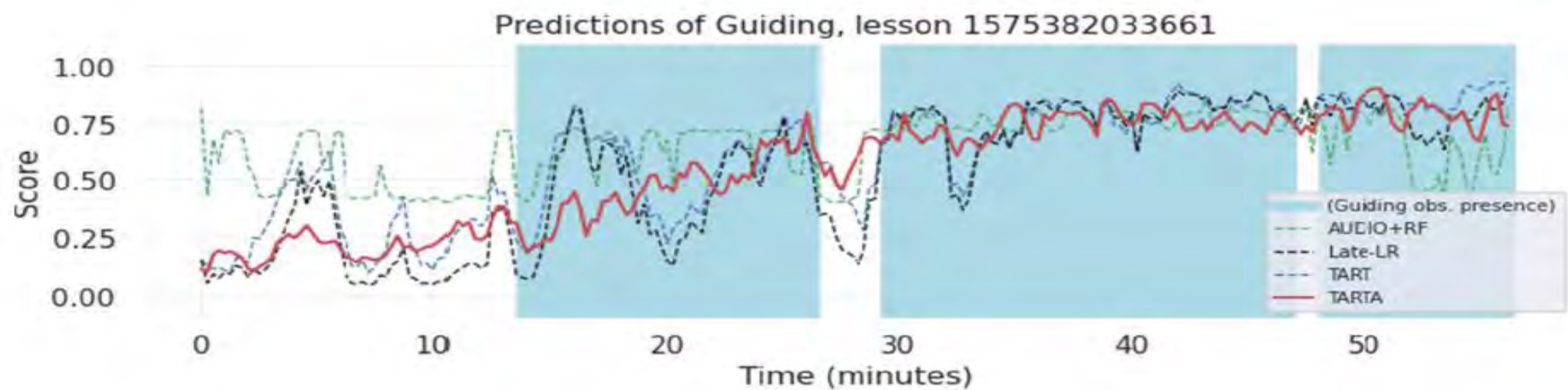
Gaze

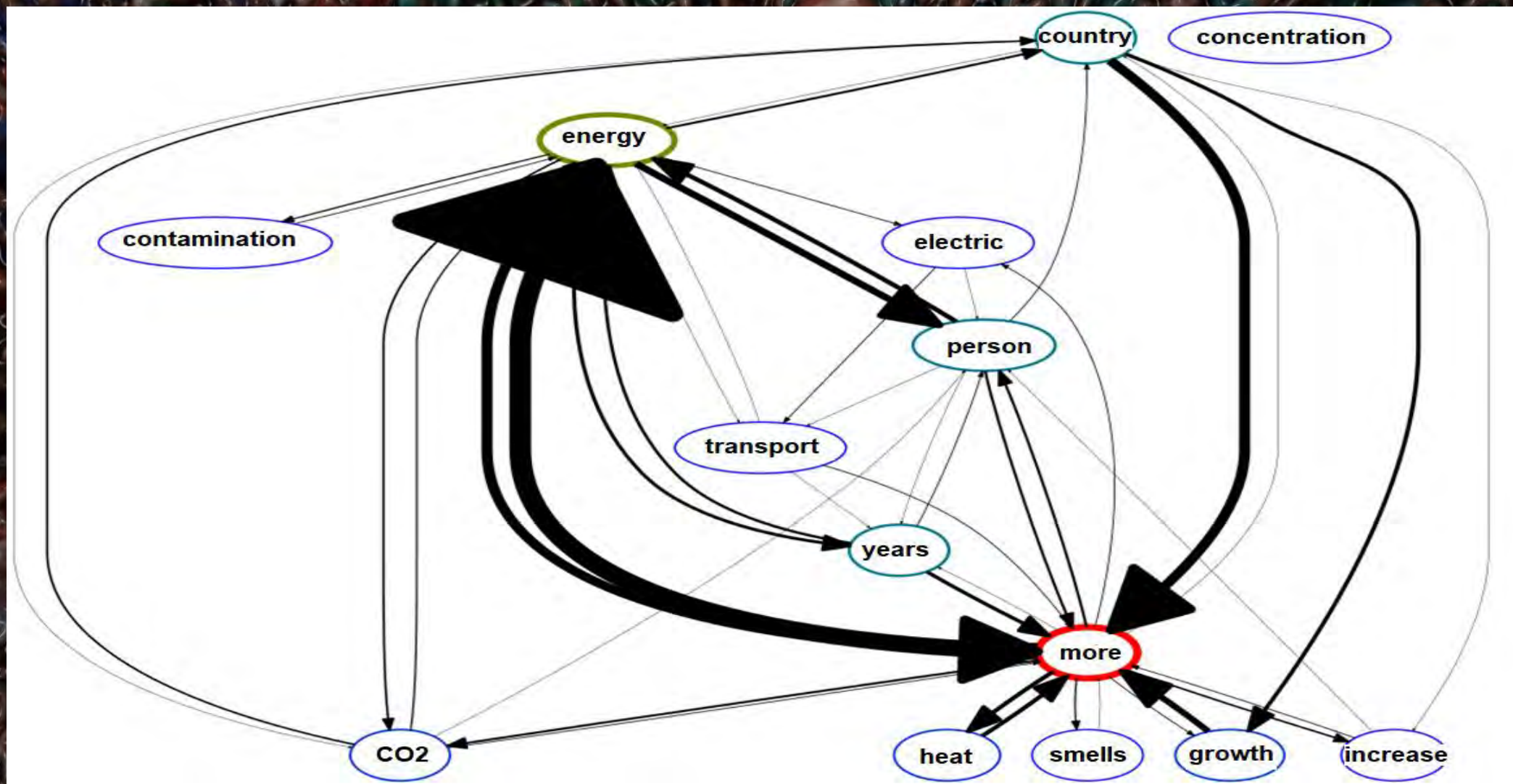
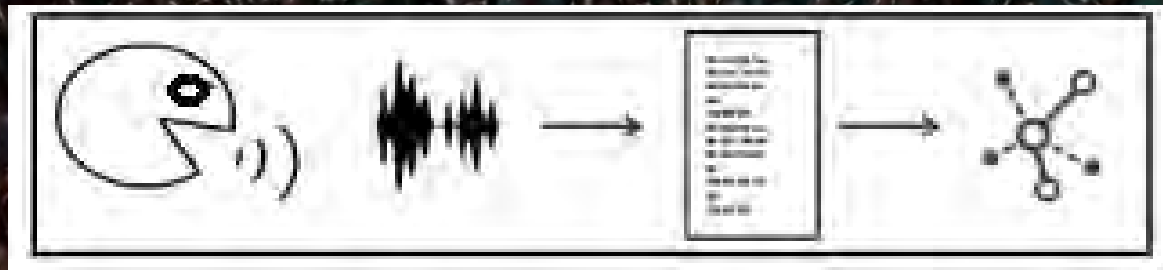
Body orientation





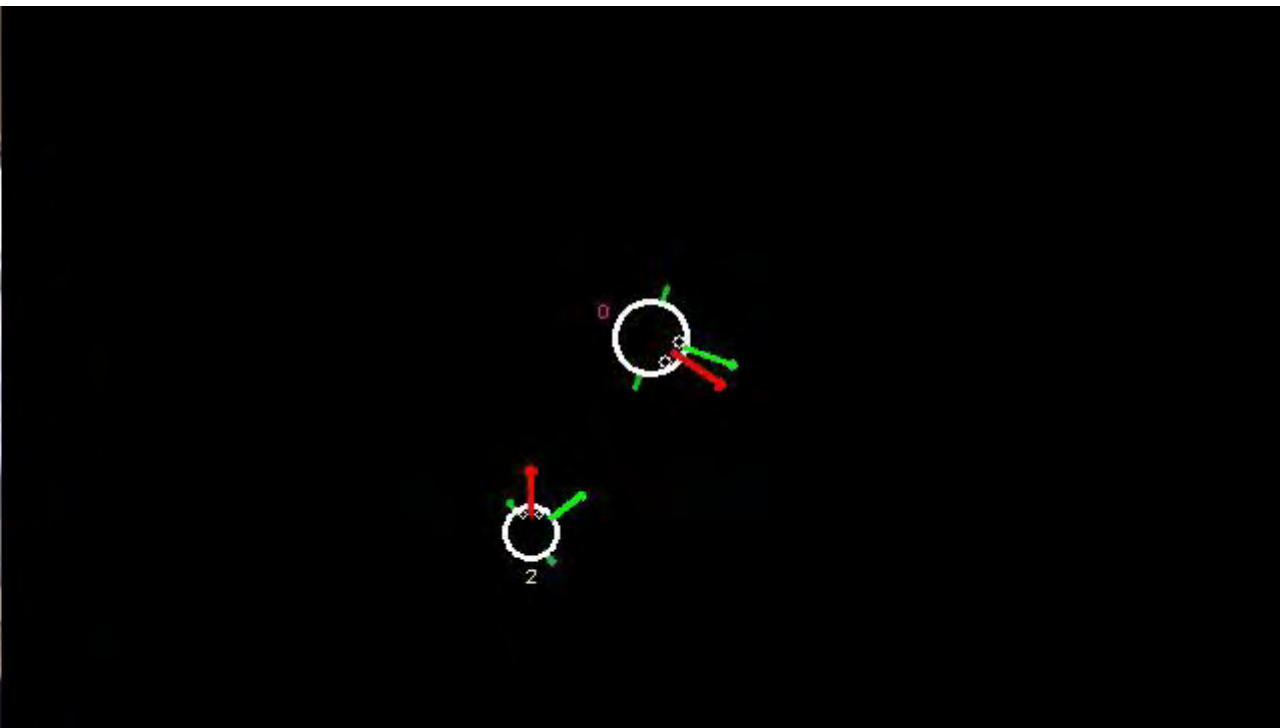
(a)





Non verbal behavior

Gaze and body orientation



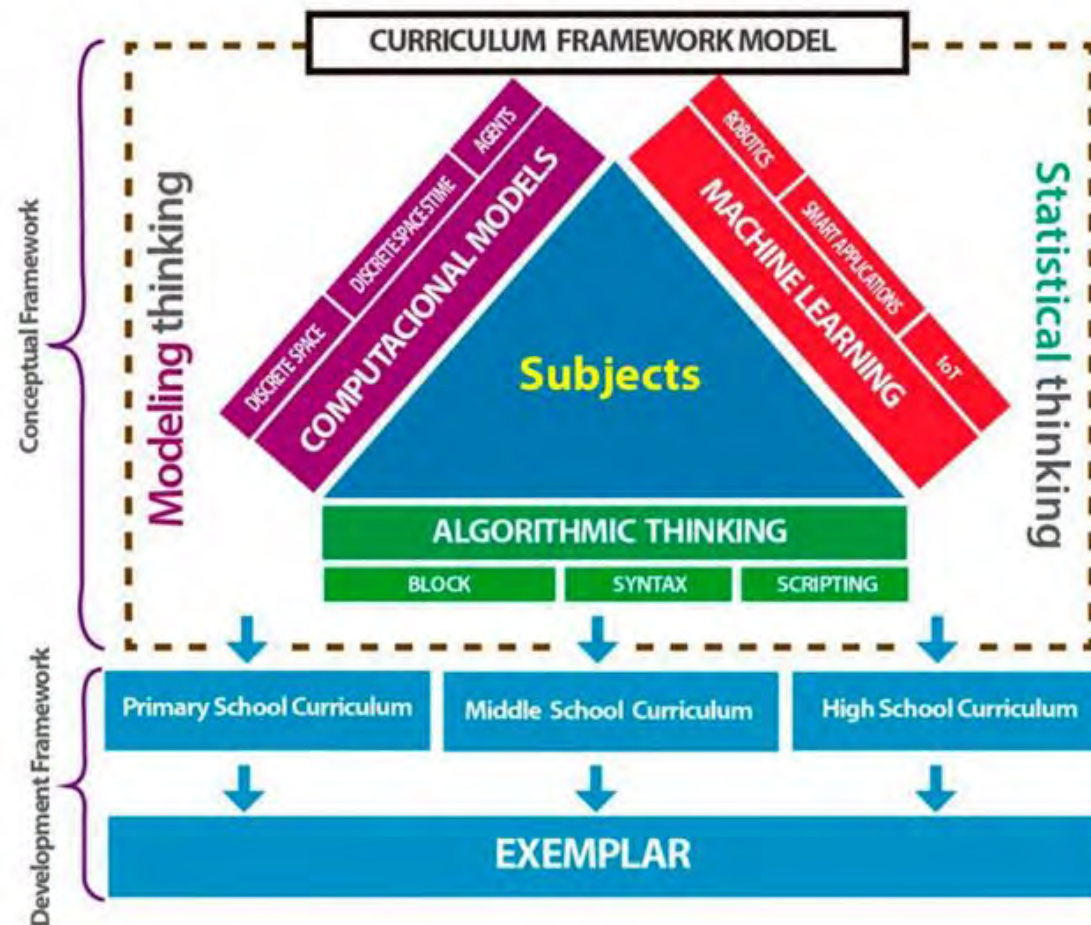


Figure 1. Curriculum framework model.

Isoda, M.; Araya, R.; Inprasitha, M. (2021) Developing Computational Thinking on AI and Big Data Era for Digital Society - Recommendations from APEC InMside I Project <https://www.apec.org/Publications/2021/03/Developing-Computational-Thinking-on-AI-and-Big-Data-Era>

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