ASSOCIATION OF MATHEMATICS EDUCATORS

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Maths Buzz

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President's Message ...Time for Reflection and Suggestions

By the time you are reading this issue of *Math Buzz*, it is quite likely that you have already completed most of the teaching and assessment tasks for the year, e.g., write pupil reports, mark PSLE papers, submit assignments for in-service courses taken, etc. Congratulations on surviving another year of teaching and/or research with its high dramas and mundane chores. It is the opportune time to reflect on your achievements (not for staff appraisal!) for the year and to take note of what you have learned and will not need to re-learn in the coming months. If you are going on holidays in Singapore or overseas, relax and rekindle your enthusiasm for the many activities put on hold while you were deeply involved in your busy work schedule. You deserve a good break with your family and buddies!

While in the Wonderland, Alice asked, "How can I know what I think until I see what I say?" If you are reflecting now, get hold of a piece of paper and jot down your thoughts as you read on. This action might result in an article for *Math Buzz*! We want to hear from you.

Recall a particularly successful mathematics lesson you conducted this year. Visualise the lesson as vividly as possible and re-create your emotional states during that lesson. Feel it, savour the sense of success and elation, "look at" the pupils with your mind's eyes, and capture their sense of wonder of being taught by a caring teacher like you. Now, write down a few adjectives to describe this visualisation. It is important to include the emotional states in our reflection.

Next, try to detach yourself from the successful lesson and examine analytically evidence or critical incidents that define why it was successful. For example, you may have asked a searching question that prompted your pupils to think about a mathematics concept relationally. Jot down the question in your reflection sheet and describe how your pupils answered it. Or, you had found the reason to explain why most of your weaker pupils kept interchanging the *x-y* coordinates, for example, (2, 3) for the point (3, 2). After they had worked through an ingenious activity you gave them, they experienced the aha! of sudden insight. At this stage of reflection, write down the specific situations and actions that define the success of your lesson. Collecting evidence to support your perception helps you to take the next step of reflection.

Now, try to see a link between the details and general principles. Do you know of some psychological or learning theories that can explain what has happened? Theories can deepen our understanding of current practice and provide inspiration for future work. The American mathematician, psychologist, inventor and actor, Charles Peirce (1839 – 1914), once wrote: "There is nothing more practical than a good theory". This explains why researchers are interested in theories and try to translate them into practice. School teachers can also benefit from a deeper understanding of education theories. Incidentally, Peirce was also noted for his work on pragmatism as a research method.

Finally, several reflection questions related to AME: What roles has AME played this year in your teaching life? Have you attended an AME talk or read something in *The Mathematics Educators* or *Math Buzz* that made an impact on your work? Do you have any suggestions for AME activities? Next May we plan to celebrate AME 10th Anniversary and welcome suggestions for this very auspicious event. Please send your comments to the editors or AME EXCO members. Ciao.

Wong Khoon Yoong
October 2003
President, AME (2002 – 2004)

Coming Attractions

10th Anniversary Celebration of AME - A Day for Mathematics Educators

AME will be celebrating its 10th Anniversary in May 2004. The celebration will take the form of a half-day event, comprising of various activities for its members. Members can look forward to fun-filled mathematical engagements, that not only challenge your wits but also thrill you to bits. So mark the month in your calendar 2004 and look out for more details in the next issue.

We would also like to place on record our appreciation to A/P Jessie Ee, Dr Yeap Ban Haur and Mr Lee Ngan Hoe for their past contributions as editors.

Contents

Some Open-Ended Activities for Teaching Primary Statistics

Ms Ng Kit Ee Dawn
National Institute of Education

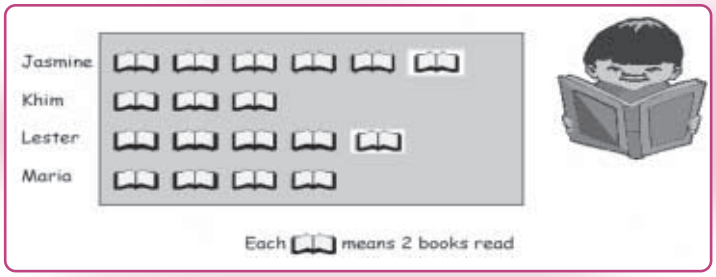
Routine exercises have been the staple of mathematics teaching and learning in many Singapore schools (Chang, Kaur, Koay & Lee, 2001). With the focus on problem solving in our Mathematics Syllabus (MOE, 2001, 10) and the increasing emphasis of interdisciplinary learning, some attention from this staple has shifted to problem solving activities and tasks which promote *active learning*, engaging pupils in meaningful negotiation of common understanding through class discussions, oral presentations and group work.

Such activities can involve practical tasks in real life problems, ranging from routine mathematical problems to problems in new contexts and open-ended investigations (Foong, 2002, 15), all attempting to engage pupils in the relevant application of concepts, skills and processes, and, at the same time, developing pupils' metacognitive abilities. This article will present three examples of open-ended primary statistical activities inspired by Foong's article (2002) where teachers can use in their mathematics classes. These activities have been explored by primary mathematics teacher trainees. Their responses to the activities and feedback on learning points will be discussed in later sections. Suggestions will also be made on how these activities can be used in class to arouse pupils' interest in mathematics and engage pupils in mathematics communication.

Activity 1: Picture Graph

Objective of Activity: Promote mathematical communication through data description.

Instructions for Activity: Write a suitable story for the picture graph below. Make your story interesting. Include as much data from the picture graph as you can in your story.



[Picture graph adapted from http://homepage.mac.com/dpsdescartes/worksheets/stat4.pdf]

This activity can be given to primary two pupils as pair or group work. The context at which data is collected has been deliberately left out so as to encourage pupils to be creative in their story telling, embedding the various data interpretations from the picture graph. More scaffolding in the form of guiding questions can be provided for weaker pupils. Instead of writing a story, these pupils can be asked to write short sentences describing what they saw on the picture graph.

Trainees teachers who tried this activity came up with interesting stories. Here is an excerpt of a response to the task taken from the work of Kandasamy *et al.* (2003):

> Once upon a time, in the southern province of China, four Confucian scholars were studying for their Imperial Exams. They studied day and night to complete the classics from the Tang Dynasty. After the Imperial Exams, the Emperor summoned them individually to his Imperial Palace...
>
> "How many books did each of you read before seating for the Imperial Exams?" boomed the pot-bellied Emperor.
>
> "12 books, your Majesty!" declared the proud Jasmine.
>
> "I read only 2 books fewer than Jasmine!" shouted the bespectacled Lester.
>
> "I only read 8 books..." squeaked Maria.
>
> Khim announced meekly, "Only 6, your Majesty..."

As discovered by the trainees, this activity not only encouraged different ways of describing the same data, it also tied in with use of language to bring cross subtle meaning, seen in the intentions behind the choice of words such as "declared", "squeaked" and "meekly" – an interdisciplinary approach.

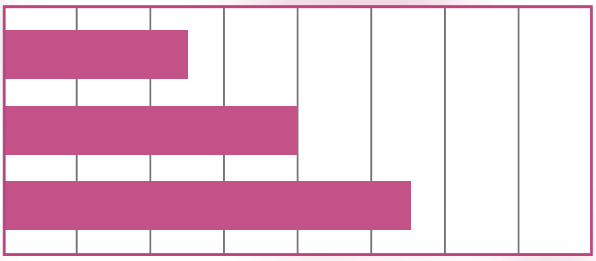
Activity2: Bar Graph

Objectives of Activity:

(i) Promote mathematical communication through data description.

(ii) Emphasize the importance of interpretation of data based on how axes are labeled and context of data collection.

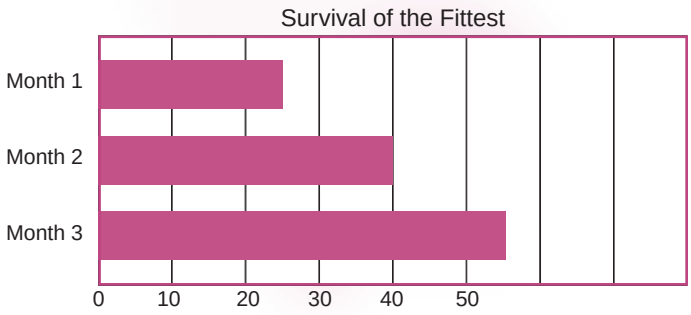
Instructions for Activity: Write a suitable story for the bar graph below. Make your story interesting. You may label the axes according to your story and give the bar graph an appropriate title. Include as much data from the bar graph as you can in your story.



[Picture graph adapted from http://homepage.mac.com/dpsdescartes/worksheets/stat1.pdf]

Such an activity can be given to primary 4 pupils. More has been left out in the statistical diagram compared to the previous activity to allow for more varied interpretations of the data. Depending on how the axes are labeled and the given title which was to summarize the purpose of the diagram, pupils can be prompted to explain why their stories "match" with the given diagram.

Below is an excerpt taken from the work of Leong *et al.* (2003):



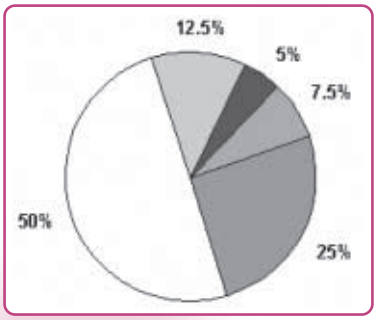
150 men were sent to the lush rainforest of the Amazon with no food and water. They were given choppers and penknives to survive in the rainforest as long as they could. The bar chart shows the number of men who left after the first, second and third month. The man who stays the longest in the rainforest and walks out of it alive will stand to win fabulous prizes from sponsors.

In trying out this activity, trainees teachers had to ensure that the numerical quantities suggested were appropriate to the context presented. There should also be a "realistic" match between data and context. This highlighted the importance of interpreting statistical diagrams critically – for not all statistics presented in the media are done so accurately and realistically.

Activity 3: Pie Chart

Objective of Activity: Highlight the role of pictorial representation of data and how statistical diagrams can affect our interpretation of a situation, especially those in the case of advertisements and promotions.

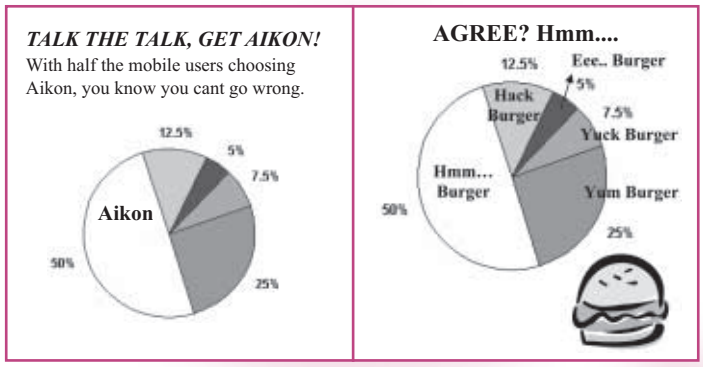
Instructions for Activity: The following pie chart shows the popularity of a product in Singapore among its competitors. Write a short advertisement selling this product making use of the statistics presented in this pie chart.



[Pie chart adapted from http://www.mathleague.com/help/data/data.htm#piecharts]

Primary six pupils who can read and interpret pie charts can be given this task. They can be encouraged to be creative while given a focused mission of selling their selected product making use of the power of statistical diagrams. Oral demonstrations of advertisements or role playing can be done in class to generate discussions on the impact of statistical diagrams on information processing and interpretation. More activities can follow where the teacher can bring in instances of misleading statistical diagrams and talk about how interpretations of such diagrams can be done critically.

Here are two samples of work done by Chua *et al.* and Yong *et al.* (2003):



In conclusion, statistics play a significant role in our daily lives. As mentioned by Pereira-Mendoza (1995), newspapers, magazines and other forms of media make use of statistical information to communicate with readers. The use of routine problems may help in promoting skills-based algorithms in the topic of statistics. However, it is also important that teachers relate statistics to its real-life applications and teach pupils to make interpretations of information presented to them in a critical, thoughtful manner. One way of doing so can be through the use of open-ended activities.

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Note: Authors of these websites are not known
http://homepage.mac.com/dpsdescartes/worksheets/stat4.pdf
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Contributions from AME at the Mathematics Synergy Symposiumn 2003

Σnrich • Σnhance • Σnergise

organised by Centre of Excellence @ Zone East on 6 September 2003

Several members of AME contributed their invaluable time and expertise at the Symposium held at Tao Nan Primary School, the headquarters of the Centre of Excellence for Mathematics (COE), on 6th September 2003. The sessions were clearly well-received by the participants. Besides the contribution of our speakers, who elucidated the audience on issues extending from teaching pedagogies to the new frontiers and identities in the educational realm, A/P Berinderjeer Kaur and Dr Tay Eng Guan also put up a display of AME materials at the symposium exhibition. AME wishes to thank the organisers for their support and cooperation.

This event has created a platform for like-minded practitioners, educators and researchers in the teaching of Mathematics to assemble pedagogical ideas amassed from primary and secondary schools. We hope that the sharing at the symposium will lead to the cultivation of more bravehearts to sally forth and experiment with new ideas and teaching strategies. Perhaps in the process of exploration, they may create more innovative ones and thereafter trigger more sharing and greater synergy in the future.

Concurrent Sessions by Members of AME

i) Excellent teaching + Excellent learning = Excellent outcomes

A/P Wong Khoon Yoong, President of AME

This interactive session had led the participants to reflect on the equation for mathematics instruction:
Excellent teaching + Excellent learning = Excellent outcomes

Different ideas about excellence in mathematics education were explored and related to simulated classroom settings. At the end of the session, participants were challenged to reflect and suggest ideas or even partake in action research for their own professional development.

ii) TIMSS-R: Performance of Singapore's Secondary Two Students in Mathematics

A/P Berinderjeet Kaur, Past President of AME

The Third International Mathematics and Science Study (TIMSS) was first conducted in 1994-1995 under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) and involved more than 40 countries. Singapore too participated in the TIMSS. A repeat of this study known as TIMSS–R, designed to provide trends in eighth grade mathematics and science achievement in an international context, was carried out in 1999.

TIMSS-R data for mathematics showed that the overall performance of Secondary Two students from Singapore was commendable and they ranked first amongst the thirty-eight countries that participated. Almost half of Singapore's eight graders were amongst the top 10% of the participants in the study.

In this concurrent session, Dr Kaur shared with the audience our students' performance on some of the 82 mathematics items that were released by IEA. The session was certainly very informative. Not only had Dr Kaur helped to shed some light on our students' strengths and weaknesses, which were of great interest and concern to our mathematics teachers, but also on some of the mathematics instructional practices adopted by our teachers that led to such an achievement.

iii) Using the Geometer's Sketchpad to Teach Geometry at the Primary Level

Dr Teong Su Kwang & Ms Ng Kit Ee Dawn

With the Singapore's Masterplan for IT in Education (MOE, 1997) now in place, the use of computers has gradually become a common feature in Singapore mathematics lessons, as a tool for integrating information and communication technology (ICT) for the teaching and learning of mathematics. Schools are now presented with a wide variety of software and internet resources for the teaching of geometry. While the use of a dynamic geometry software such as the Geometer's Sketchpad (GSP) is recommended in the secondary school mathematics curriculum, it is not integrated into the primary school mathematics curriculum.

In the concurrent session of the symposium, a framework where some GSP tasks could be used to enhance the teaching and learning in the primary mathematics classrooms was also discussed. These GSP tasks were classified by levels of ICT use, ranging from highly structured student participant tasks to free explorations. Several examples of these tasks were also shared by Ms Dawn Ng while Dr Teong Su Kwang facilitated the spontaneous question and answer session at the end of the presentation.

iv) Mathematical Problem Solving

Mr Joseph Yeo Kai Kow

In teaching mathematical problem-solving, teachers appear to use one or two strategies to solve a mathematical problem in the classroom. Although some teachers have explicitly encouraged students to use varied strategies to solve mathematical problems, students continue to solve problems using one or two strategies. Research has also shown that good novice problem solvers are unable to solve problems in more than one way.

In this session, Mr Joseph Yeo offered his audience insights on how different types of mathematical problems could be solved using varied strategies. The sharing had met with very positive feedback from primary and secondary Mathematics teachers alike and many had expressed interest in attending more of Mr Yeo's future sharing sessions.

Olympiad Training for Teachers on 4 & 5 September

by Mrs Tan-Foo Kum Fong, National Institute of Education

Contributed by Mrs Juliana Ng
Master Teacher

Teachers in the West 7 Cluster felt the need to upgrade their skills in Mathematics Olympiad-type questions and expressed interest in learning. The conceptualization of the idea led to 2 half-day workshops organized by Master Teacher, Mrs Juliana Ng, at the W7 Cluster Mathematics Resource Centre situated at Nan Hua Primary School.

For two days, 4 and 5 September 2003, teachers were handheld in solving past years' Olympiad questions on various types: unfamiliar, non-routine and challenging. Despite the grueling and demanding moments during the two 3-hour, hands-on sessions where teachers challenged themselves to solve Olympiad type problems, the sessions were very well received.

Not only had Mrs Tan-Foo walked the participants through questions from the topics on combinatorics, logical thinking and number theories in a very encouraging and stimulating manner, she also encouraged teachers to share what they had gathered at the workshop in their schools. Besides the formal delivery at the workshop, various books brought by Mrs Tan were also displayed to show participants the resources they could use to extend their students' knowledge and help their students make connections in real life situations.

The sessions were indeed successful and fruitful.

Thank you Mrs Tan for the invaluable sharing. Your enthusiasm and positive attitude towards the learning of Mathematics is truly infectious and inspiring.

Helping Young Children Overcome Difficulties with Maths

A/P Pamela Sharpe, National Institute of Education

The workshop was designed to alert participants to the key features of early numeracy development in young children. It was based on the premise that children who are numerate and confident in using and applying their knowledge of numbers in preschool will have few problems with school mathematics later on. Hence the workshop focused on the sequence of competencies young children naturally develop and the kinds of adult support vital for this successful process in mathematics. A central thread was the rationale behind the newly developed numeracy guidelines contained in the joint MOE/NIE produced Framework for a Preschool Curriculum in Singapore entitled: Nurturing Early Learners. The emphasis was on revisiting early steps and strides in the numeracy development sequence, the foundations of early mathematical knowledge and understanding. Examples were provided of the kinds of activities adults might†engage in with children to both boost their confidence and accommodate their early misperceptions. A stress on oral work, speaking, lsitening and engaging in number stories and sentences was the major theme.

4 x 3: Is it 4 + 4 + 4 or 3 + 3 + 3 + 3?

Dr Ng Swee Fong, National Institute of Education*

The above question was raised by a parent in a letter to the editor column ("4 x 3", 2003). That such an elementary mathematical concept as multiplication could arouse such a sense of confusion and frustration in adults, who most probably could reel off the multiplication tables one to twelve without giving a second thought to the task, seems bizarre. However the problem faced by the letter writer was not whether she could recall the multiplication facts, but rather the different meanings associated with the multiplication symbol. She was puzzled why her child's answer of 3 x 4 to a picture showing four plates with three apples on each (see Figure 2) was marked wrong. The writer was perceptive when she noted that whilst multiplication is commutative, i.e. in multiplication order does not matter for higher mathematics, it nonetheless mattered in the case of her child's work at primary one.

This paper discusses the different interpretations associated with multiplication. Two interpretations of multiplication are developed in the Singapore primary mathematics curriculum. The concept of multiplication as repeated addition is developed in primary one. In primary two, children are provided with activities to help them "see that multiplication is commutative" (CPDD/MOE, 2000, p. 44). I will draw specific examples from the texts and teacher's guide written by the Curriculum Planning and Development Division of the Ministry Of Education to illustrate how the Singapore primary mathematics curriculum develops the first two interpretations of multiplication. Multiplication as a Cartesian product is not included in the Singapore curriculum but this will be the last interpretation to be discussed in this paper. In each case I will tease out the different expressions associated with the multiplication symbol. Implications for teaching multiplication are discussed in the concluding section of this paper.

Some different interpretations of multiplication

Multiplication as repeated addition

The purpose of teaching multiplication as repeated addition to children is to provide children with an economical substitute for an extended count; i.e. "by counting in groups instead of in ones" (Churchill, 1961, p. 147).The Primary Three Teacher's guide (CPDD/MOE, 1999, p. 31) gives the following clarification of meanings that could be associated with the multiplication symbol.

> In the symbolic language, the following two conventions are used by writers, teachers and children to write multiplication of numbers:
>
> 3 x 4 means '3 multiplied by 4', i.e. 3 + 3 + 3 + 3
>
> 3 x 4 means '3 times 4' i.e. 4 + 4 + 4.

Mathematical symbols, presented as words or marks on papers, are themselves attached to abstract mathematical ideas (Skemp, 1989). The words and phrases that are first used to talk about these ideas and hence the attached symbols invariably shape how individuals communicate these ideas. In the example cited above, the words and phrases could be described as language patterns associated with the symbol of multiplication. Because of their abstractness, writers tend to make known the meaning they choose to attach to the multiplication symbol. In the discussion of multiplication in his book *The Psychology of Learning Mathematics,* Skemp (1986) highlighted the fact that, for consistency, a x b should be read as 'a multiplied by b' in his book. He acknowledged that a x b could be read either as 'a multiplied by b' or 'a times b'.

Skemp provided pictorial representations to show how 'a multiplied by b' is different from 'a times b'. If 5 x 3 is read as '5 multiplied by 3' then the first diagram matches this expression. The second diagram, however, is represented by the expression as 5 times 3. Because Skemp was clear which interpretation he was attaching to the symbol x , readers are able to follow his discussion.

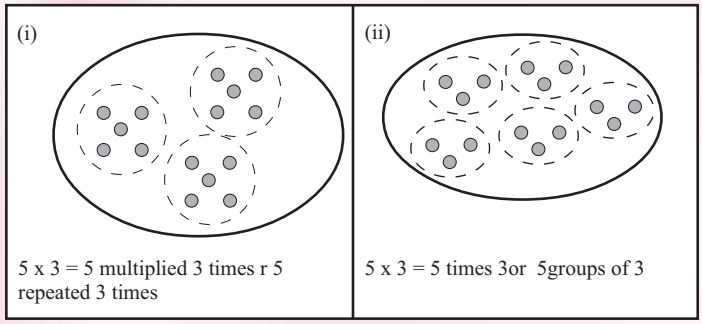


Figure 1

Applying MOE's clarification, in (i), 5 + 5 + 5 = 5 multiplied by 3 = 5 x 3

In (ii) 3 + 3 + 3 +3 + 3 = 5 times 3 = 5 x 3

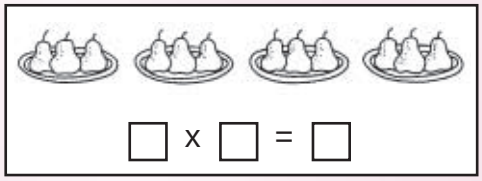


Figure 2

Hence when provided with the example in Figure 2 (CPDD, Workbook 1B, 1994, p. 76), depending on their interpretation of the x symbol, children can choose to write either 4 x 3 or 3 x 4. Both responses are equally valid.

The array interpretation of multiplication – rows and columns

In Singapore, the array interpretation of multiplication is introduced at primary three and the commutative property of multiplication is developed through this interpretation (Teacher's Guide, CPDD/MOE, 1999). The following example is provided in the Primary Three Teacher's Guide (1999, p. 32).

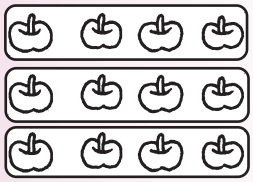


Figure 3: 3 groups of 4

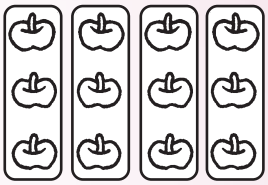


Figure 4: 4 groups of 3

In the rectangular array the arrangement of the apples could be interpreted as 3 groups of 4 or 4 groups of 3. Since both 3 times 4 and 4 times 3 describe the same structure, therefore 3 x 4 = 4 x 3. Also the reader could choose the word multiply to describe the relationship between the rows and columns. In this case it would be

4 + 4 + 4 = 3 + 3 + 3 + 3
or in words
4 multiplied 3 = 3 multiplied 4

The commutative property of multiplication emerges only from this particular structure and not from the first interpretation of multiplication.

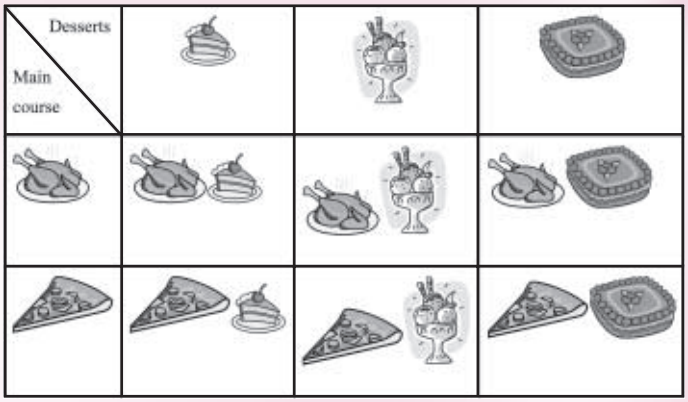
Multiplication as a cartesian product

How many different ways can you order your lunch if you can have a main course and a dessert?

Menu	
Main course *Chicken rice* *Pizza*	Dessert *cake* *ice-cream* *fruit tart*

***The author thanks Martyn Quigley for helpful discussion and suggestions in the process of preparing this paper.**

One way is to use a matrix to list all the possible ways.



Given two choices for main course and 3 choices for dessert, the total number of ways a meal could be ordered is 6, where 6 is the product of 2 and 3. By providing children with similar examples, children can abstract the operation underpinning such tasks. Likewise if there are m ways to choose one item and n ways for selecting another, then there are m x n ways for selecting these two items. In this interpretation, the multiplication operation is neither seen as repeated addition nor as an array. The answer happens to be the product of the numbers provided.

Symbols and meanings

The preceding section shows that symbols, on their own, have no meanings. It is only when contexts are provided do the symbols acquire meanings. What is more confusing is that different words could be used for the same symbol. Often the meanings of the different words are lost when the focus of an activity is at getting a numerical answer where no context is provided – for e.g. when a 3 and a 4 are linked together with the multiplication symbol x , the number on the other side of the equal sign should be 12. At primary one, the concept of multiplication is introduced to children through activities which are based on meanings. However what meanings symbols mean to these children will not be clear until they use words to describe them. The words children use depend on the adults teaching them. If the adults themselves are not clear about the meanings of the words they use, their ambiguity may be a source of confusion for the children.

Would it be easier for children if the word 'times' is used for the multiplication symbol? It might help alleviate the problem initially. However the teacher may have difficulties modelling the situation of 45 times 3. Also would it be educationally sound to restrict children to only one interpretation of the multiplication symbol?

Implications for teaching

To an adult this discussion might seem trivial. However it is important to understand the world of children. It would be a blow to children's confidence if the multiplication facts they had put so much effort into learning was deemed wrong.

For the sake of consistency, authors of textbooks choose to use a specific meaning for the multiplication symbol in their work. However their choice does preclude the development of other meanings that are equally valid. For teachers to know the intent of the authors, far more knowledge of the subject matter is called for in teachers who continually need to understand the mathematical ideas themselves, their development and their interrelation. Therefore teachers need to exercise professional judgement in assessing the development of concepts presented in textbooks and this involves scrutiny of materials and the choice of words used in books. Because teachers are aware that symbols on their own have no meanings, they therefore need to choose which words they want to use to develop a particular meaning of multiplication. Once they have made their decision, they need to be consistent in their use. However if children's written symbolic responses do not match the teacher's, the children's written responses, which showed correct multiplication facts, should be accepted. Teachers could interview children to find out the meanings children attach to the symbol, if any. It may be a case that children were recalling multiplication facts they were taught whilst in kindergarten or by their parents who were teaching the facts purely through rote.

The moral of the story is that teachers need to be clear about the meaning of the symbol used and teachers must be consistent in the usage. However teachers need to know that there could be other equally valid meanings besides their own attached to the symbol.

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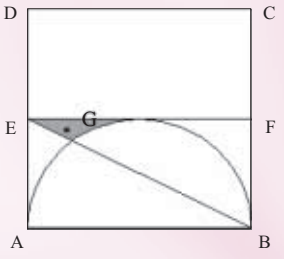
Acute Triangles in a Square

The following problem was communicated by Dr Tang Wee Kee, National Institute of Education

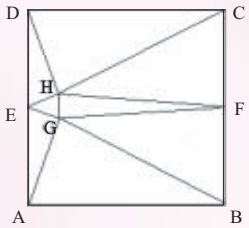
Given a square, can we cut the square into a finite number of acute triangles. An acute triangle is one with every angle less than 90 degrees.

Solution contributed by Mr Albert Wong, Temasek Polytechnic, A/P Lee Peng Yee, National Institute of Education

Here is one such solution. In the following diagram, EF cuts the square ABCD into two equal halves. Pick a point G in the shaded region as shown below.



Then we can see that < AGB is an acute angle since G lies outside the semi-circle. If G lies on the circle it is known that < AGB would be a right-angle. Note also that < AGE is an acute angle. If G lies below the line BE, < AGE would be an obtuse angle (that is, greater than 90 degrees). Then join G to the points A, B, E, and F. Reflect G about the line EF, and we obtain H. Join H to C, D, E, F, and G as shown below.



We obtain 8 acute triangles. Hence we have cut the square successfully into 8 acute triangles.

To cut the square into acute triangles, we must put one point or more points inside the square. Further, for every point inside the square we must attach 5 edges to the point otherwise we would have at least one triangle with an angle greater than 90 degrees. It is easy to see that we cannot form acute triangles with only one point inside the square. With two points inside the square, the minimum number of triangles (acute or otherwise) we can form is 8. Since we have done it above, hence we can say that the square can be cut into a minimum number of 8 acute triangles.

We can also cut the square into 10, 11, 12 acute triangles and indeed for any finite number greater 12. We could not do it for 9. Hence we conjecture that it is not possible to cut the square into 9 acute triangles.

FORTHCOMING EVENTS FOR YOUR ATTENTION!!

	Talk / Workshop	Speaker	Audience	Date & Time	Venue
1	Teaching Mathematics with CAS	B Kutzler (Austrian Center for Didactics of Computer Algebra, ACDCA) & V Kokol-Voljc (University of Maribor, Faculty of Education, Slovenia)	Teachers, teacher trainers, math education researchers, decision makers (from school authorities)	21 Oct 2003, Tuesday 2.30pm – 5.30pm	LT 12C, SP
2	Problem-Solving Items in PSLE	Dr Yeap Ban Har, NIE	Primary teachers, parents of upper primary pupils	10 Jan 2004, Saturday 9am – 12noon	MLT1, SP
3	How to Score Distinction in the 2004 'O' Level Additional Maths Exam	Mr Ong Ee Heng, SP	Students, parents and teachers	6 Mar 2004, Saturday 9am – 12noon	MLT1, SP
4	Keys to Effective Mathematical Problem Solving for Primary Teachers	Mr Joseph Yeo Kai Kow, NIE	Primary mathematics teachers	13 Mar 2004, Saturday 9am – 12noon	NIE
5	Keys to Effective Mathematical Problem Solving for Secondary Teachers	Mr Joseph Yeo Kai Kow, NIE	Secondary mathematics teachers	20 Mar 2004, Saturday 9am – 12noon	NIE
6	Rethinking the Values and Ways of Teaching Algorithms in School Mathematics	Dr Fan Liang Huo, NIE	All mathematics teachers	Eleventh Annual General Meeting cum AME 10th anniversary	To be determined
7	Beyond Problem Solving – Reasoning in the Mathematics Classroom	Dr Berindeerjeet Kaur, NIE	Secondary mathematics teachers		
8	The Mathematics of the Public Holidays of Singapore	Dr Helmer Aslaksen, NUS	All mathematics teachers		
9	Integrating New Assessment Strategies into Mathematics Classroom	Dr Fan Liang Huo, NIE	All mathematics teachers	3rd / 4th quarter 2004	To be determined

- Though the targeted group of audience is specified, other mathematics educators who are interested are also welcomed.
- Please refer to our website http://math.nie.edu.sg/ame/ for more information.

Contribution Invited

Please send to:

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