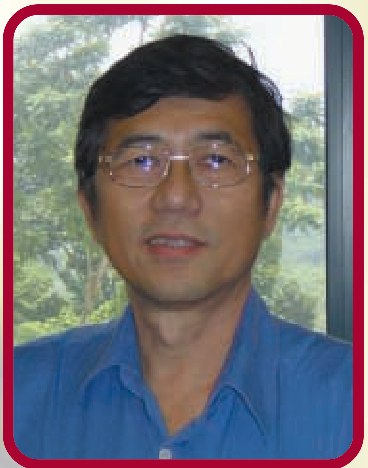
ASSOCIATION OF MATHEMATICS EDUCATORS
Website: http://math.nie.edu.sg/ame/

Maths Buzz

Issue 5 Vol.2 MITA (P) No. 312/08/2003 May 2004

President's Message.. *Thank You*



For the past two years, it has been a great honour to serve as AME President. I have been fortunate to work with a wonderfully committed Executive Committee, supported by competent part-time clerical staff and enthusiastic volunteers. It is time for AME to have a new President to lead our Association to greater heights. I wish to take this opportunity to thank my Exco members for their full support. During this period, the Association has also jointly organised several activities with the Department of Mathematics and Science, Singapore Polytechnic, and the Singapore Mathematics Society, and I am grateful for their strong collaboration. I must not forget to thank the Association's most important partner, the Mathematics and Mathematics Education Academic Group of NIE, as most of the Exco members are from that group! These joint efforts have helped to promote mathematics education among our members and the public, and such collaboration could be further strengthened for mutual benefits. Indeed, the Association should explore partnership with schools, Junior Colleges or Institutes.

Another reason I feel truly grateful is the learning opportunities, serving as President. This involves chairing meetings, making decisions in a frantic way, representing the Association, writing these messages, and so on. These roles cannot be learned through reading or watching on the sidelines; they have to be practised and tested in real, sometimes under quite stressful, situations. As we aim to promote the process aspect of learning among our pupils in project work and service learning, we as teachers must find new situations to sharpen our own skills in completing various processes and roles. A particularly effective way to achieve this is to hold responsibilities in a society like AME! Hence, I welcome on board those who wish to be elected as members of the next Executive Committee. The election will be held at the Eleventh Annual General Meeting cum Tenth Anniversary Celebration on 29 May. We have organised several interesting talks for you, with exhibitions of books and display of teaching materials.

January 9 this year was a memorable day for the community of mathematicians and mathematics educators in Singapore. On this day we celebrated the 65th birthday of Professor Lee Peng Yee.

Professor Lee was the "millennium" President of AME from 2000 – 2001, irrespective of whether one defines the beginning year of the third millennium to be 2000 or 2001! On this joyful occasion, AME presented Professor Lee with a small gift to express our gratitude for his invaluable contributions to AME. For those who love to solve mathematics puzzles, check that 65 is the second number which can be expressed as the sum of two squares in two ways, and it is the magic constant of a n by n magic square. What is the value of n?

Wong Khoon Yoong
April 2004
President, AME (2002 – 2004)

Contents

Solving Ancient Chinese Mathematics Problems – The Chinese Approach versus the Algebraic Method

Dr Ng Wee Leng and Miss Lee Wing Kei Joyce

The past decades have seen a resurgence of interest in problem solving as an integral part of the mathematics curriculum. Early Chinese mathematical works, well known for their emphasis on solving real-life problems, are an invaluable source of mathematical problems suitable for the teaching and learning of primary and secondary mathematics.

In this article, we shall discuss an intriguing problem known as the "Problem of One Hundred Fowls" in the Chinese classic *Zhang Qiujian's Mathematical Manual* (Zhang Qiujian Suanjing), making some fundamental comparisons between the Chinese approach to solving this problem and the algebraic method which our students are familiar with.

Early Chinese mathematicians solved problems with multiple solutions from as early as the fifth century A.D and "The Problem of One Hundred Fowls" is one such example. To better understand the Chinese approach, we shall begin with a related problem known as "The 100 Monks and 100 Buns Problem" which was first recorded in *The Systematic Treatise on Arithmetic* (Suanfa Tongzong) written during the Ming dynasty (1368-1644).

Problem 1: The 100 Monks and 100 Buns Problem

100 monks eat 100 buns. If each senior monk eats 3 buns and 3 junior monks share 1 bun, how many senior monks and junior monks are there?

Most students are likely to solve this problem by first forming simultaneous equations as follows:

Let x be the number of junior monks and y, the number of senior monks. Then

$x + y = 100$ ——————(1)

$\frac{1}{3}x + 3y = 100$ ——————(2)

Solving the equations, we obtain $x = 75$ and $y = 25$.

How did the Chinese solve it then? Several different methods have been found in various Chinese classics in mathematics. We shall present two different methods as provided in the Chinese classics, *Master Sun's Mathematical Manual* (*Sunzi Suanjing*) and *The Systematic Treatise on Arithmetic*.

Method 1

This approach was given in *Master Sun's Mathematical Manual*.

Let's triple the number of buns each monk eats. That is, 1 senior monk eats 9 buns and 3 junior monks eat 3 buns. So total number of buns 100 monks eat is 300. Now note that

1) Number of buns each junior monk eats = 1
2) Number of buns each senior monk eats = 1 + 8
3) Total number of buns = Total number of monks + 200

Number of senior monks is therefore

= 200 ÷ 8

= 25

Finally, number of junior monks

= 100 – 25

= 75

Hence the number of senior monks is given by

$(100 \times 3 - 100) \div (3 \times 3 - \frac{1}{3} \times 3) = 25$

Method 2

The Systematic Treatise on Arithmetic presents a slightly different approach. In contemporary notations, it states that the number of senior monks is 100 ÷ (3 + 1) = 25 and the number of junior monks is 100 – 25 = 75.

Why is that so? Note that since one senior monk eats 3 buns and 3 junior monks eat 1 bun, a group of 1 senior monk and 3 junior monks (4 monks in total) eat 4 buns. Since there are 100 buns, the number of such groups is 100 ÷ (3 + 1) = 25. Finally, in each such group, there is one senior monk. So the number of senior monks equals the number of such groups.

Using the above ideas, students can now solve Problem 1 using the Model Method without having to solve simultaneous equations which is not in the primary mathematics syllabus. We leave it to the readers to work out the details.

We are now ready to solve the Problem of One Hundred Fowls.

Problem 2: Problem of One Hundred Fowls

A rooster costs 5 wen, a hen costs 3 wen and 3 chicks cost 1 wen. If 100 wen can buy exactly 100 fowls, how many roosters, hens and chicks are there?

Note that in ancient China, a *wen*, a copper coin, was the lowest denomination in the currency.

Again, students are likely to solve this problem by forming simultaneous equations as follows:

Let x be the number of roosters, y, the number of hens and z, the number of chicks. Then

$x + y + z = 100$ ——————(1)

$5x + 3y + \frac{1}{3}z = 100$ ——————(2)

Since there are 3 unknowns but only 2 (linearly independent) equations, there are more than one solution to this problem.

Eliminating z from the equations, we obtain

$7x + 4y = 100$ ——————(3)

From (3), we can deduce that x is a multiple of 4. The respective values of y and z can then be found by substituting multiples of 4 into x as follows:

	a	b	c	d	e	f
x	4	8	12	16	20	...
y	18	11	4	-3	-10	...
z	78	81	84	87	90	...

Since x, y and z must be positive integers, we can see from the table above that there are 3 different solutions to this problem as stated in columns (a), (b) and (c).

Many students may find certain parts of the above method difficult to follow. So how did the Chinese solve the problem? The hint given in *Zhang Qiujian's Mathematical Manual* is as follows:

Whenever you add 4 roosters,
subtract 7 hens and add 3 chicks.

This hint does not tell us how the problem can be solved but rather the inter-relationship between the variables. The Chinese referred to it as "increase-decrease ratio" (*zeng jian lü*).

Observe that in applying this "ratio", the sum of the number of roosters and chicks added is equal to the number of hens subtracted, and so the total number of fowls remains the same. In addition, since the cost of 4 roosters and 3 chicks is equal to the cost of 7 hens, the total cost of the 100 fowls remains 100 *wen*.

With the "increase-decrease ratio", all we need now is one set of numbers that satisfy the conditions given in the problem.

Assuming that the number of roosters is zero, Problem 2 becomes:

A hen costs 3 wen and 3 chicks cost 1 wen. If 100 wen can buy exactly 100 fowls, how many hens and chicks are there?

Notice that the modified Problem 2 is equivalent to Problem 1, the *100 Monks and 100 Buns Problem.* We therefore know readily that there are 25 hens and 75 chicks.

Of course, we still have not solved Problem 2 because we assume that the number of rooster is zero. But all we need to do now is apply the "increase-decrease ratio" and obtain the following table.

Case	No. of roosters	No. of hens	No. of chicks
1	0 +4	25 -7	75 +3
2	4	18	78
3	8	11	81
4	12	4	84
5	16	-3	87

We have therefore found, as in the earlier algebraic method, the 3 different solutions for Problem 2.

In the next issue of Maths Buzz, we shall examine other methods the Chinese had used in solving this problem. We shall also explain how the "increase-decrease ratio" can be derived.

Century Years – Leap or Common!!!

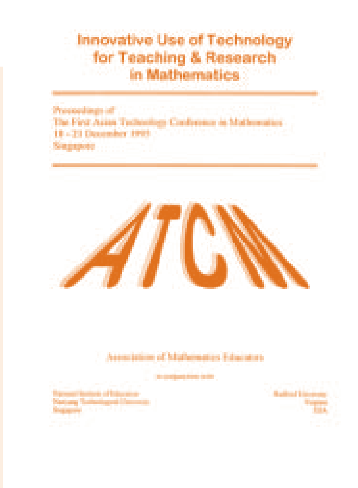
Dr Dindyal Jaguthsing

It is common knowledge that in the Gregorian calendar a leap year has 366 days and a common year has 365 days. Leap years are exactly divisible by 4, whereas common years are not. This follows from the fact that one solar year is taken to be 365.25 days and so to make up for the additional day, every four years we have a year with 366 days. However, this does not apply to century years, **why?**

Teachers need to clarify this with their students. The problem is that one solar year is not exactly 365.25 days; it is 365.2422 days or 365 days, 5 hours, 48 minutes and 46 seconds. This implies that our estimate of 365.25 days for a solar year overestimates the actual solar year. Over time, this overestimation of 0.0078 days accumulates to a substantial amount of one full day every 1/0.0078 ≈128.2 years. Hence, every 128.2x3 = 384.6 years (≈400years) the calendar would have been overestimating by three full days. To take care of this situation, the Gregorian calendar reduces the number of days by three every 400 years. To do this, the strategy has been to count century years as common years if they are not exactly divisible by 400. Thus, the century years 1700, 1800, and 1900 were common years because they are not exactly divisible by 400, whereas the year 2000 was a leap year.

Readers will note that 384.6 years is taken to be 400 years in the approximation. What similar problems are we going to face in the *not so near* future?

The 9th Asian Technology Conference in Mathematics



Proceedings of the1st ATCM held in Singapore in 1995

The Asian Technology Conference in Mathematics (ATCM) has its origins in Singapore when the National Institute of Education (NIE) hosted the inaugural ATCM in 1995 at the Bukit Timah campus. Since then, the ATCM has received international recognition and the annual conference has been hosted by countries such as Australia, China, Japan, Malaysia, Taiwan and Thailand.After nine years, ATCM will once again be in Singapore. ATCM 2004 will be held at the NIE campus in Jurong from **13 to 17 December 2004**. With the theme, "*Technology in Mathematics: Engaging Learners, Empowering Teachers, Enabling Research*", ATCM 2004 promises to be a conference for everyone, from mathematics educators and researchers to mathematics teachers at every level.

This conference will focus on examining the best practices of applying technology in the teaching and learning of Mathematics and in Mathematics research. In particular, the conference will explore how technology can be exploited to enrich and enhance Mathematics learning, teaching and research at all levels. The conference will cover a broad range of topics on the application and use of technology in Mathematics research and teaching. These include, but are not limited to:

- o Mathematics for Information Technology
- o Geometry Using Technology
- o Internet and Web Technology for Mathematics
- o Graphics Calculators
- o Computer Algebra Systems
- o Mathematical Research using Technology
- o Mathematical and Statistical Software and Tools on the Web
- o Learning and Assessment using Technology

The tentative list of plenary speakers for ATCM 2004 includes:

- o Xia-shan GAO (Mathematics Mechnization Research Center, Academia Sinica, China)
- o Peter FLYNN (University of Melbourne, Australia)
- o KOH Thiam Seng (Director, ETD, MOE, Singapore)
- o Richard NOSS (University of London, Institute of Education, England)
- o Katsuhiko SHIMIZU (Tokyo University of Science, Japan)

In addition to the usual paper presentations, ATCM 2004 will, for the first time, include poster presentations. This is to cater to participants who prefer to use posters as a mode of presenting their work or projects. Poster presentations from students are welcome as well.

As is the tradition at ATCM, there will be workshops and tutorials. These include workshops on TI calculators, Casio calculators, graphing tools, and problem-solving with IT. These workshops are conducted by experts in their relevant field and have always been a popular feature of ATCM.

Submission of abstracts for paper or poster presentation is now open at the conference website. Please visit: http://www.atcminc.com OR http://math.nie.edu.sg/atcm for details.

For enquiries, please contact: atcm2004@nie.edu.sg

On Permutation and Combination

Dr Toh Tin-Lam and contributions from participants of his workshop on Permutation and Combination

Most students find the topic, *Permutation and Combination* difficult because solving the problems from this topic requires both conceptual and procedural understanding.

A workshop was conducted for secondary school teachers at the National Institute of Education during Term 1, 2004. Each course participant was asked to contribute different questions that have the same answer of $^{10}C_2$. Appended below are some interesting questions contributed by the participants.

The questions are classified into two different categories: (1) questions that require direct application of knowledge and (2) questions that require interpretation before their answers can be obtained.

A: <u>Questions that require direct application of knowledge</u>

<u>Question 1</u>: A class committee consisting of 2 members is to be selected from 10 nominees. How many ways can they be selected?

***Remark*:** This is a direct question testing the students' knowledge of ^nC_r. Compare Question 1 with the following question:

<u>Question 1a</u>: A class chairperson and vice-chairperson are to be selected from a group of 10 nominees. How many ways can they be selected?

Note that Question 1a can be extended from Question 1. However, the answer for Question 1a is $^{10}C_1 \times {}^9C_1$ instead of $^{10}C_2$.

<u>Question 2</u>: Find the number of ways in which 10 students can be divided into two groups consisting of 2 and 8 students.

<u>Question 3</u>: A committee comprising of 4 members is to be selected from 12 people. How many ways can such a committee be formed if the oldest and youngest person must be chosen?

***Remark*:** Question 3 is slightly more difficult than Question 2 but it is still a "direct question". Students are required to know that since the oldest and the youngest are to be included in the committee, the remaining two committee members can be selected from the remaining 10 people. Hence there are $^{10}C_2$ ways of forming the group.

Question 4 appended below is similar to **Question 3**:

<u>Question 4</u>: A bowling club has 12 members which include 2 sisters. How many ways can 4 girls be selected to represent the club in a competition if the 2 sisters must be included in the team?

<u>Question 5</u>: In how many ways can 10 newborn babies in a hospital be returned to their mothers with exactly one error made? (i.e. exactly two mothers receive the wrong babies)

***Remark*:** It may not be easy for students to observe that the answer to **Question 5** is $^{10}C_2$, as it needs slight interpretation.

In this case, the condition of the problem needs to be re-examined. Exactly two mothers receiving the wrong babies, is identical to considering the number of ways of selecting two out of the 10 mothers to receive the wrong babies.

<u>Question 6</u>: Let f be the linear function $f(x) = ax + b$, where a and b are integers from 0 to 9 with $a > b$. How many functions are there?

***Remark*:** The above questions needs some interpretation: Now, a and b are two different integers from the set {0,1,2,3,4,5,6,7,8,9}, so there are 10 digits from which we need to choose two digits. The order $a > b$ is already fixed. Hence once two digits are selected, the choice of a and b is already determined. Therefore there are $^{10}C_2$ ways of performing the task of selecting two digits.

B. <u>Questions that need interpretation</u>

<u>Question 7</u>: Ten points lie on a circle. How many different chords can be constructed on the circle using these points?

***Remark*:** Students should re-interpret the construction of a chord as equivalent to choosing two points.

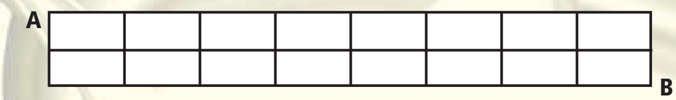
<u>Question 8</u>: Given that the set B = {135, 141, 145, 152, 163, 171, 185, 191, 197, 199}, find the number of subsets of B such that the sum of the elements in each subset is between 200 and 400.

***Remark*:** Students need to recognize that forming subsets to satisfy the criterion of **Question 8** is the same as forming a subset with two elements.

<u>Question 9</u>: Find the number of different ways to arrange 8 red balls and 2 green balls in a row.

***Remark*:** At the first glance, one may think that **Question 9** is not in the syllabus since it involves permutation of objects, some of which are indistinguishable. However, it can be regarded as a selection problem where two out of 10 boxes, are selected to contain the two green balls. It would then automatically imply that the remaining 8 boxes be used to contain the remaining for the red balls.

<u>Question 10</u>:



A man wants to move from Point A to Point B. At each junction, he can only choose to move south ↓ or east →. How many ways can he move from A to B?

***Remark*:** The question involves traveling 10 steps from A to B with each step either an east (E) movement or a south (S) movement. There are 2 "S" and 8"E" to be arranged. Hence, it goes back to the case of **Question 9** and the answer is $^{10}C_2$.

<u>Question 11</u>: I have 9 cans of Coke and 2 cans of Sarsi. How many ways can I arrange these 11 cans of drink in a row such that the 2 cans of Sarsi are always separated?

***Remark*:** The nine cans of Coke give rise to 10 spaces between any two of them, and hence two spaces need to be chosen for the two cans of Sarsi. The answer is obviously $^{10}C_2$.

Compare **Question 11** with **Question 11a** below:

<u>Question 11a</u>: 9 cadets and 2 officers are to pose for a photograph. In how many ways can they be arranged such that the two officers are not standing next to each other?

Remark: The answer for this question is not $^{10}C_2$ (since it is unreasonable to assume that human beings are indistinguishable!)

<u>Question 12</u>: There are 11 antennas of which 9 are functional and 2 are defective. Assuming that all of the functional and defective antennas are indistinguishable, how many ways can the antennas be arranged such that no two defective antennas are consecutive?

***Remark*: Question 2** is very similiar to **Question 11**.

<u>Question 13</u>: A teacher wants to buy 11 pens from a bookshop which sells blue, red and green pens only. Given that the teacher buys at least 1 pen of each colour, how many ways can the teacher buy the 11 pens?

It will be an interesting exercise for the reader to explain why the answer to **Question 13** is $^{10}C_2$.

Conclusion

Instead of overwhelming students with problems on Permutation and Combination that require them to consider many complicated cases and conditions, teachers can consider giving problems with "simple" and neat answers but require students to interpret the questions. By beginning with questions that need direct application of knowledge, students can consolidate their basic knowledge on the topic. As the demands of the questions increases, students whose basics are firmly entrenched can use their higher order thinking skills to meet the challenges of these "difficult" questions.

Power of Counterexamples

Ng Swee Fong
National Institute of Education

Chan Siew Sharn, Betty
Marsiling Primary School

Often children assume that since triangles have three sides, therefore *any* three sides will form a triangle. Such a misunderstanding is possible because of the way children are taught geometry. Often their encounters with triangles are with those shown in textbooks. For example it is quite normal for children to be set work to find areas and perimeters of triangles presented in two-dimensional forms. The question whether these triangles they are asked to work with exist does not arise because they must as they are offered by textbooks. Furthermore teachers themselves often do not doubt the existence of such triangles. Therefore if teachers teach what is in the textbooks, why should children query the existence of any triangles. Therefore it is 'normal' for children to assume that any three sides will form a triangle. That some teachers have similar beliefs is not surprising as they have gone through similar learning experience and their belief gives credence to van Hiele's theory of geometric thought that learners learn about shapes and their attending properties as a result of the methods used and the content of instruction – and not because of maturation. What this means is that if learners have not engaged with activities where they explore the conditions which ask which three sides will form a triangle, they come away from their learning with the misunderstanding that any three sides will form a triangle. Suitable activities must be offered to help learners construct the appropriate understanding of triangles. This article reports the result of such an activity carried out with a group of middle to high ability primary five children.

These children have been taught triangles but they have yet to learn how to classify triangles into the three different groups - equilateral, isosceles and scalene. The following question was posed to these primary five children. The aim of the activity was to find out the children's knowledge of triangles.

> *Suppose you are given two sets of the following straws: 3cm, 5 cm 10cm. Using any three of the six straws each time, and if you could re-use the straws, how many different triangles can you form?*

First the children were asked to conjecture as to how many triangles they could form with the above dimensions. After discussing among themselves, some children conjectured that they could make six triangles and while others hypothesised that there were nine triangles. Two triangles were the same if they each have the same dimensions but different orientations, otherwise they were different. The children were asked to record their triangles. The following examples listed in Figures 1 and 2 show the different types of triangles these children thought could be formed given the above dimensions. Children's responses showed that they had the belief that any three sides would form a triangle and this was not surprising given the nature of their learning experiences with geometry.

Pause for thought

- How are the triangles in Figure 1 different from those in Figure 2?
- Can you see which set of triangles are impossible to make?

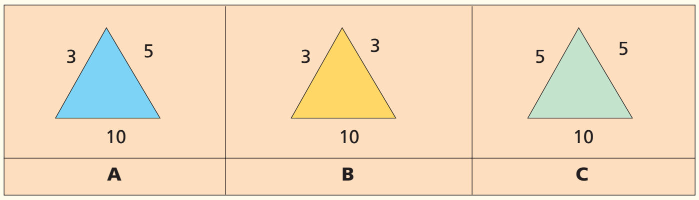


Figure 1

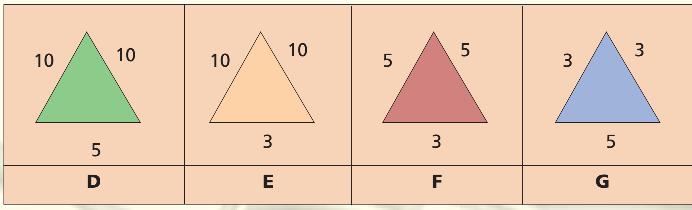


Figure 2

These children were then given two sets of straws of lengths 3cm, 5cm and 10 cm. They were asked to check if these straws could make the triangles they had conjectured. The triangles in Figures 3 and 4 constructed using straws correspond to those triangles Figures 1 and 2 respectively. Much to the surprise of the children, they found that only four different triangles were possible and these were triangles D, E, F and G in Figure 4. The constructed examples in Figure 3 were counterexamples to challenge children's conjectured triangles in Figure 1.

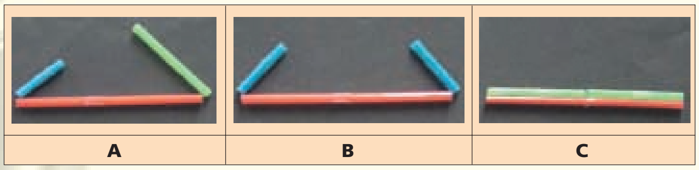


Figure 3

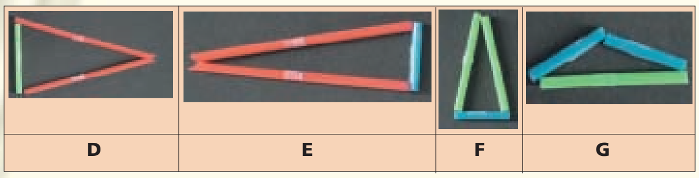


Figure 2

red straws – 10 cm, blue straws – 3 cm and green straws – 5 cm.

The next task was to use the examples of triangles and counter-examples to ascertain what relationship exists between the three sides of these triangles. After examining the dimensions of the constructed triangles against the counterexamples, the children came up with the following observation: *the sum of the lengths of any two sides must be more than the third side.*

Will this rule apply to any triangles? Here onwards children can discuss whether this rule will apply to any triangle. They can do this by reasoning about the lengths of sides or they could use software such as the Geometer's Sketchpad to explore this relationship.

The above lesson shows how powerful it is to tease out children's misconceptions by first asking children to offer their conjectures about triangles and then provide them with concrete activities to test their conjecture. Such a simple activity allows children to engage with the heuristic – make a supposition. Also children learn about the power of using counterexamples to disprove a conjecture. The conclusion that can be drawn from this one lesson is that these children acquired far more than just the knowledge about the relationships of the three sides that form triangles. These children learn about how to make and test conjectures and the power of counterexamples – activities engaged by mathematicians but seldom by children in mathematics classrooms.

Problem-Solving Items in PSLE

A Seminar by Dr Yeap Ban Har, National Institute of Education, Singapore

Ban Har began the seminar by emphasising the importance of helping children develop conceptual understanding of the mathematics they learn, visual spatialisation and estimation skills as well effective metacognitive abilities.

Ban Har used items from past PSLE to help illustrate the points he made. He categorised the PSLE items into three different categories. These are listed as follows.

Items that included extraneous information: Such test items included extraneous information which meant that pupils had to make decisions and select appropriate data to answer these items.
Items that tested estimation skills: For example, pupils were asked to estimate the size of an angle.
Items that tested higher-order thinking skills: The following PSLE item tested higher-order thinking.

1 + 2 + 3 + 4 +... + 96 + 97. When the first 97 whole numbers are added up, what is the digit in the 'ones' place of this total? (Q15, PSLE 2000)

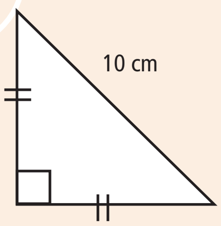
One possible solution is as follows:
Pair the numbers 3 with 97, 4 with 96 and others, so that they can form sums of 100. Because there is an odd number of terms in the series 3 + 4 + ... + 97, the number 50 does not have a partner to pair with to form the total of 100. Hence, the sum of the digits in the ones place is 1 + 2 = 3.

Although the task appears to require a great deal of tedious computation, a good problem solver would use an alternative heuristic to complete the task.

Because pupils often modelled their teachers' problem-solving behaviours, Ban Har emphasised that teachers demonstrate to pupils how they metacognise about a given problem, reflecting the hows and whys that allowed them to solve that problem. One purpose of such reflection is to enable pupils to consider alternative strategies to the same item. Also, it is just as important for pupils to reflect why a certain strategy resulted in a wrong solution. It is vital for pupils to learn from their errores as well as from their success.

Ban Har concluded the seminar by offering the audience this thought-provoking question - **How can a primary school pupil solve this problem?**

Find the area of this isosceles triangle



Reported by Dr. Kwek Leong Chuan

Making Meaning/Making Sense in Mathematics Classrooms

Workshop with Leone Burton, Professor of Mathematics Education, UK.

Leone began by spelling out her agenda for the morning. She wanted the participants to concentrate on three things, engaged activity, social context and reflection. She introduced us to what she meant by these, and then she showed some video excerpts of students working in classrooms.

Engaged Activity: this is mathematical activity which is purposive and involved. It challenges pupils to search, exchange, explain, make conjectures, explore and evaluate (as well as many other things). A lesson for the classroom, therefore, is that demands on pupils must be consistent with their drive to make meaning. Another lesson for the classroom is that speaking comes before reading and writing and we therefore have to build speaking about purposive activities into our classroom activities. So we need a social context.

Social Contexts are both collaborative and discursive. Within collaborating groups, pupils learn to try things out, to conjecture, to explore, to justify, evaluate and convince others of their findings. Collaborating groups, therefore, use language not only for speech but also for reading and writing.

Reflection: this is the space within which we make connections to what we already know and build networks of knowing. Reflection requires contexts that are familiar but also provides the teacher with feedback on the learning of their students.

Leone then focused on a long list of different kinds of talk which have been identified in classrooms in the UK. The list began with 'telling' but continued with narrating, reporting, informing, summarizing, questioning, explaining, exploring, suggesting, conjecturing, testing, speculating, persuading, collaborating, imagining, evaluating, arguing, reflecting, specializing, generalizing, expressing feelings. Participants were given a matrix with these kinds of talk down the side and across the top were asked: *Does this happen in your classroom? Who does it? Often? rarely? What is the context?* As Leone ran down the list, we were invited to complete the matrix for ourselves answering for when the different kinds of talk were to be found in our classrooms.

We then watched 3 video excerpts. The first was a Scottish teacher working with a class of 12 year olds discussing their perceptions of a mathematical poster. The second was a Welsh teacher of 5-6 year olds playing a logic game with attribute blocks. The third was 2 nine-year-olds in England using bottle tops to support their work on fractions. No teacher was around. Again, Leone provided us with a matrix with the different kinds of talk down the side, and across the top the questions When does this happen in the video? With whom? What is the context? And the learning? In small groups we discussed what we had observed in the video.

In conclusion, Leone gave us 5-10 minutes to summarise our learning from the morning into a number of key points, to identify our unanswered questions, and to put these in order of priority. She warned us that only one key point or key question would be expected from each group and no point or question was to be repeated. So we had to listen very closely to what everyone said and judge if our point or our question had been addressed. For those teaching in secondary or tertiary education, this is an alternate way of dealing with fixed lecture time. Instead of presenting lectures as the conventional style, lecture notes can be handed out to groups of students in advance of the lecture. They can be asked to work through the lecture notes and prepare a list of questions or key points, prioritise them and the lecturer's response to these forms the basis of the class. This causes the class to be more interactive, students feel more closely involved, they have done work out of class on the topic and they relate to the questions and points raised and answered. For the lecturer, their preparation for the class becomes printed text and they are placed in the situation of having to think on their feet in response to what is raised by their students. They learn what works for the students and what does not giving them an opportunity to improve their class preparation.

Reported by Dr Ng Swee Fong

ERAS Conference 2003 — My Learning Experiences

The Educational Research Association of Singapore (ERAS) was established in 1986 as a non-profit organisation to promote the cause of educational excellence in Singapore and the region via the conduct, promotion and use of relevant educational research. AME has forged close ties with ERAS. Last year, AME sponsored five of its members to attend the ERAS Conference 2003.

Here is the reflection of Mr. Yeo Tiong Bin, one of the five members of AME, sponsored to attend the ERAS conference. Mr. Yeo is a retired principal who is currently an Education Consultant in the Chinese Development Assistance Council (CDAC)

The ERAS Conference on "Research In and On Classroom" was held from 19th to 21st November 2003. There were altogether three keynote lectures and eleven concurrent sessions.

Here are some key points extracted from some papers, which are relevant to my area of work.

Developing and fostering higher-order thinking in the classrooms
Associate Professor Carol K. K. Chan of The University of Hong Kong first examined the current beliefs about higher-order thinking and discussed related issues and problems. She highlighted some important points:

- embedding higher-order thinking in the school curriculum
- roles teachers could play in promoting higher-order thinking
- knowledge building in the school curriculum

Does formal assessment in primary science support the TSLN vision?
Dr Boo Hong Kwen of National Institute of Education highlighted a number of questions that do not really test pupils' thinking skills. She then made the following suggestions:

- include opened-ended segments in Multiple Choice Questions by using 2-tier questions
- test higher order thinking, e.g. by designing investigative activities
- minimise the use of factual questions
- consider alternative perspectives such as in items & mark schemes
- broaden the mark scheme for supply type items
- introduce other modes of assessment, e.g. Project Work (PW)

Parental involvement in children's mathematical learning
Mr Lee Ngan Hoe of National Institute of Education revealed that parental involvement was one of the contributing factors to the success of children's mathematical learning. In the school setting, he suggested parents could play the role of resource persons, partners in the journey of learning and consumers of pupils' creative work. In the home setting, parents could work as partners in pupils' homework. He felt that parents could also engage pupils in meaningful mathematical learning while helping out in household chores and at leisure.

Qualities of good mathematics teachers
Dr Berinderjeet Kaur of National Institute of Education found the attributes of a good mathematics teacher obtained from the perspectives of our pupils in the local primary and secondary schools similar to the Standards for Excellence in Teaching Mathematics in Australian schools.

Some of these characteristics are:

- committed and enthusiastic professionals who continue to extend their mathematical knowledge as well as their student's learning
- Setting high and achievable goals for themselves and their students.
- committed to the continual improvement of teaching strategies
- ability to arouse and sustain pupils' interest in mathematical learning.

After attending the conference, I would like to propose the following changes in my organisation:

- improve the existing structure for the staff training programme in my organisation

- Initiate programmes to improve pupils' writing and oral skills in the English Language as the papers on IPMA (International Project on Mathematical Attainment) study and TIMSS (Third International Mathematics and Science Study) revealed that pupils generally lacked communication skills

- Initiate collaboration amongst teachers to share teaching strategies, knowledge and innovative ideas

Contributed by Mr. Yeo Tiong Bin

Birthdays on the 29 February!

What is the probability of anyone around the world having their birthdays on Feb 29, assuming that it is equally likely for a person to be born on, any days of a year?

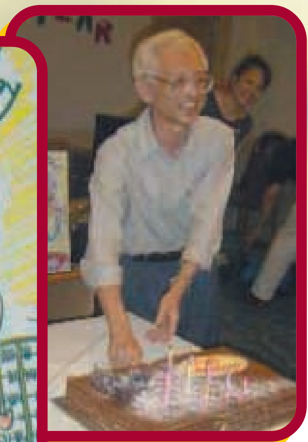
According to Professor Louis Chen, director of the National University of Singapore's Institute for Mathematical Sciences, the probability is one in 1 461. In other words, out of every 1 million people, there about 684 leap-year babies, which is extremely rare.

(Straits times, 8 February 2004)

FORTHCOMING EVENTS FOR YOUR ATTENTION!

No.	Talk / Workshop	Speaker	Date & Time	Venue	Audience
1	A Plenary Talk: Lessons from Mathematics Classrooms around the World	Dr David Clarke, University of Melbourne	Eleventh Annual General Meeting cum AME 10th Anniversary 29 May 2004 Plenary session 0900 – 1010 hrs Concurrent Sessions 1120 – 1230 hrs	NIE 7-01-LT4	All mathematics teachers
	Mathematical Problems and Mathematical Inquiry	Dr Peter Pang, National University of Singapore			All mathematics teachers
	Rethinking Issues about Teaching Algorithms in School Mathematics	Dr Fan Lianghuo, National Institute of Education			All mathematics teachers
	The Mathematics of the Public Holidays of Singapore	Dr Helmer Aslaksen, National University of Singapore			All mathematics teachers
	Primary-Level Mathematics Activities Featured in Math Buzz	Dr Douglas Edge, National Institute of Education			All mathematics teachers
2	Integrating New Assessment Strategies into Mathematics Classroom	Dr Fan Lianghuo, National Institute of Education	25 September 2004, Saturday 9.00am – 12.00pm	To be confirmed at a later date	All mathematics teachers

Celebration of Prof Lee's 65th Birthday



To celebrate our dear Prof Lee Peng Yee's 65th birthday, MME organised a syposuium cum dinner on Friday, 9 January in honor of his contributions to NIE and the AG. A mini-symposium on Integration Theory spanning from morning till evening was organised in LT3. A number of international academics who were Prof Lee's former students and collaborators from as far as the Czech Republic, such as Professor Stefan Schwabik, to the Philippines and The People's Republic of China, came to join in the celebration and pay tribute to Professor Lee Peng Yee, for his enormous contributions as a teacher and mentor to many in the field of mathematical research. They presented their research findings at the symposium and shared the latest developments in the field of Integration Theory, setting the directions for future work. A dinner was held in the evening at the NTU, Mayflower Corner to complete the entire celebration.

Contribution Invited

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Design, Colour Separation & printed by Oxford Graphic Printers Pte Ltd