ASSOCIATION OF MATHEMATICS EDUCATORS
Website: http://math.nie.edu.sg/ame/

Maths Buzz

Vol.8 Issue 2 MTICA (P) No. 076/08/2007 October 2007

President's Message...

Processes go beyond thinking skills and heuristics in the School Mathematics Curriculum 2007. A significant added emphasis manifested in the framework of school mathematics curriculum implemented this year has been in the component of processes. This component has expanded to include:

1) Reasoning, communications and connections; and
2) Applications and modeling.

The intended curriculum issued by the Ministry of Education, exemplifies the above as follows:

1) Reasoning, communication and connections

- **Mathematical reasoning** refers to the ability to analyze mathematical situations and construct logical arguments. It is a habit of mind that can be developed through the application of mathematics in different situations and contexts.

- **Communication** refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their own understanding of mathematics and sharpen their mathematical thinking.

- **Connections** refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life. This helps students make sense of what they learn in mathematics.

Mathematical reasoning, communication and connections should pervade all levels of mathematics learning, from primary levels to the advanced-levels.

2) Applications and modeling

- **Applications and modeling** play a vital role in the development of mathematical understanding and competencies. It is important that students **apply** mathematical problem solving skills and reasoning skills to tackle a variety of problems, including real-world problems.

- **Mathematical modeling** is the process of formulating and improving a mathematical model to represent and solve real-world problems. Through mathematical modeling, students learn to use a variety of representations of data, and to select and apply appropriate mathematical methods and tools in solving real-world problems. The opportunity to deal with empirical data and use mathematical tools for data analysis should be part of the learning at all levels.

As the framework for the school mathematics curriculum theoretically drives the pedagogy of the teacher, what does all this mean to us, teachers?

In our classrooms, many of us are highly dependent on textbooks to implement the curriculum. Are the textbooks addressing these emphases? We need to examine our textbooks critically and if need be reframe some of the mathematical tasks to meet our goals.

A typical mathematics teacher's classroom pedagogy is comprised of numerous D-S-R [Demonstration-Seatwork-Review] instructional cycles. How can we recast our pedagogy to facilitate reasoning and communication in the classroom; use of real world examples to formulate mathematical models and make connections between mathematical topics as well as mathematics and other subjects of the school's curriculum?

In addition to the workshops and in-service courses conducted by CPDD at MOE to help teachers equip themselves with the necessary knowledge and skills to facilitate the "processes" in their lessons, the association too has been proactive in addressing them. Through our last three mathematics teachers conferences, we have provided teachers with many introductions on how to address these emphases. Next year, the theme of our conference is: Mathematical Problem Solving. We hope that you will join us for an enriching and enjoyable day on 29th May 2008.

A/P Berinderjeet Kaur
President
Association of Mathematics Educators
Nanyang Technological University

Contents

Contributions Invited

Please send to:
The Editor
Maths Buzz c/o Mathematics & Mathematics Education
National Institute of Education
1 Nanyang Walk Singapore 637616
Email: foo_kum_fong@moe.gov.sg

Mathematics Teachers Conference 2007 1st June 2007 @ NIE

Foo Kum Fong, Ministry of Education
Ng Swee Fong, National Institute of Education

The Mathematics Teachers Conference 2007 carrying the theme *Mathematical Literacy* was jointly organized by the Association of Mathematics Educators and the Mathematics and Mathematics Education Academic Group, National Institute of Education. Singapore. The theme of this year's conference is Mathematical Literacy. A total of 808 local teachers and 84 local and foreign mathematics educators participated in this one-day annual conference.

Altogether there were 22 concurrent sessions by practitioners, educators and researchers at the following levels: 11 Primary, 8 Secondary and 3 Junior College. Professor Jin Akiyama of Tokai University Education Development Laboratory, Japan, was the guest lecturer of the conference. He gave two lectures, in each he showed how the various 'mathematical toys' in his collection could be used to teach mathematics. There were six other keynote speeches addressing the theme of the conference. Dr Fong Ho Kheong, the founding president of Association of Mathematics Educators talked about the "*What, Why and How*" of Mathematics literacy. In his keynote "*Relating Literacy to Mathematical Learning*", Professor Douglas Edge addressed the issue of change in four specific areas: content, process, technology and communication. In their talk, "*Mathematics literacy in the case of quantitative reasoning*" Professor Liu Yan and Mr Luis Tirtasanjaya Lioe argued that it was important for teachers to study and understand the what constitutes quantitative reasoning and how to teach it. Professor Ng Swee Fong talked about multiple "*literacies*" of representations in the case of the Model Method and letter symbolic algebra. The importance of reading and writing to learn mathematics at Junior college and higher levels of learning was discussed by Professor Tay Eng Guan. Professor Berinderjeet Kaur with Ms Jenny Loong provided insights on mathematics literary and competence of 223 Junior College students.

The conference was a success because it brought together practitioners from diverse background, giving participants a chance to meet and exchange ideas on how to improve the teaching and learning of mathematics. The conference organisers look forward to bringing practitioners together once more in Mathematics Teachers Conference 2008.

The Converse of Pythagoras' Theorem

Lin Shilin Charlene and Leong Yew Hoong
National Institute of Education

The converse of Pythagoras' Theorem (PT) has recently been included in the Secondary Two Express Mathematics syllabus. In the syllabus, this is spelled out as "determining whether a triangle is right-angled given the lengths of three sides" (MOE, 2007). Although students are not required to know the full meaning of "converse", it is useful to give students a general idea of the concept. In addition, it strengthens the pedagogical content knowledge of teachers in the process of planning a lesson on this topic. In this article, a range of approaches is suggested to introduce students to the concept of "converse". But before we embark on this, we thought it might be useful to give an overview of the meaning of "converse" in the context of other related terms.

1. Overview of the meaning of "converse"

Consider the statement: "if **p** is true, then **q** is true".
The **converse** of this is "if **q** is true, then **p** is true".
The **inverse** is "if **p** is **not** true, then **q** is **not** true".
The **contrapositive** is "if **q** is **not** true, then **p** is **not** true".

Example:

	Given		Conclusion
Pythagoras' Theorem	Triangle is a right-angled triangle	⇒	The sum of the squares of the two shorter sides of the triangle *equals* the square of its longest side
Converse of Pythagoras' Theorem	The sum of the squares of the two shorter sides of a triangle *equals* the square of its longest side	⇒	Triangle is a right-angled triangle
Inverse of Pythagoras' Theorem	Triangle is **NOT** a right-angled triangle	⇒	The sum of the squares of the two shorter sides of the triangle is **NOT** equal to the square its longest side
Contrapositive of Pythagoras' Theorem	The sum of the squares of the two shorter sides of a triangle is **NOT** equal to the square its longest side	⇒	Triangle is **NOT** a right-angled triangle

Figure 1: Pythagoras' Theorem and its converse, inverse and contrapositive

Notice that the "*converse*" and "*inverse*" statements are in fact the *contrapositive* of each other. In addition, a given statement and the contrapositive of it are equivalent statements. For example, if the following statement is true:

"If a triangle is a right-angled triangle, then the sum of the squares of the two shorter sides <u>equals</u> the square of the longest side."

Then, logically, the following statement is also true:

"If the sum of the squares of the two shorter sides of a triangle is <u>not equal</u> to the square of its longest side, then the triangle is <u>not</u> a right-angled triangle."

It is important to note that if a theorem is true, the *converse* of the theorem need not necessarily be true. They have to be proven as separate theorems. Before tackling a problem, one should be clear of "what is given" and "what is the conclusion". This ensures that the appropriate, valid theorem is applied. In the next section, we use a number of examples to illustrate the idea of "converse" as well to highlight the need to check if the converse of a known theorem is true.

2. Illustrating to students the meaning of "converse"

(i) Start with a "natural" example that is non-mathematical

Stating the "converse" using a more intuitive example that students can better relate with reduces the abstractness of the concept.

Example:

	Given		Conclusion	Validity
	I am a human being	⇒	I am a mammal	True
Converse:	I am a mammal	⇒	I am a human being	? (False)

Going one step further, we can prompt students with the question: "Is the converse true of false?" This makes students aware that the converse of a statement is a different statement from the original statement and that its validity must be tested.

(Ii) Use mathematical examples that students are already familiar with

Introducing mathematical examples of "converse" gives students a taste of "converse" in the field of Mathematics. This sets the stage for the presentation of the converse of PT subsequently.

Example 1:

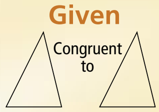
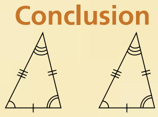
	Given		Conclusion	Validity
	m and n are even integers	⇒	$(m + n)$ is an even integer	True
Converse:	$(m + n)$ is an even integer	⇒	m and n are even integers	? (False)

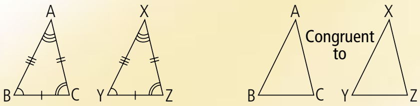
Once again, pose the question: "Is the converse true or false?" After eliciting students' responses, a counterexample may then be presented to disprove the converse:

$m = 3$ and $n = 5$, then $(m + n) = 8$ [even] but m and n are **odd**

We see that for the sum of two integers to be even, the addends need not necessarily be even. This shows that the converse is false.

Example 2:

Given		Conclusion	Validity
 ΔABC ≡ ΔXYZ	⇒	 "All corresponding sides and angles are equal"	True



Converse: "All corresponding sides and angles are equal" ⇒ ΔABC ≡ ΔXYZ ? (True)

Similarly, pose the question: "Is the converse true of false?" The topics "Pythagoras' Theorem" and "Converse of Pythagoras' Theorem" are situated after the topic "Similarity and Congruency" in the Secondary Two syllabus. Thus, this example can serve as a recapitulation of students' prior learning.

3. Presenting the converse of Pythagoras' Theorem

Students who are new to the concept of "converse" may find it difficult to give the converse of PT in written form. A diagrammatic form is suggested as shown in figure 2 below. This idea of presenting a diagrammatic overview of proofs is derived from Leong (2007).

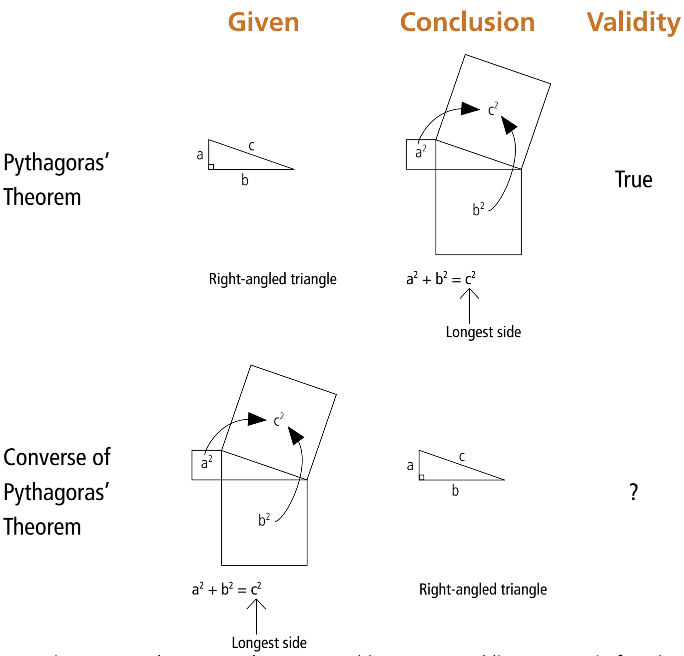


Figure 2: Pythagoras' Theorem and its converse (diagrammatic form)

4. A sample investigative activity

An investigative approach is suggested to allow students to explore the validity of the converse of PT for themselves. Using the *Geometers' Sketchpad* (GSP) software, an interactive IT activity may be designed. A sample is shown below:

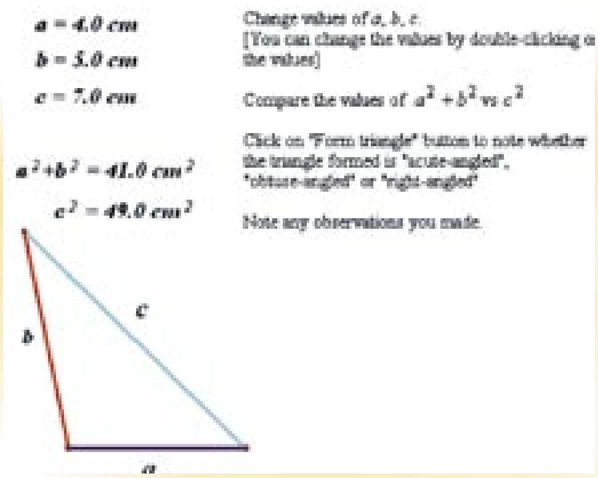


Figure 3: Capture of a possible IT activity designed using GSP

Scaffolding the investigation:

(1) Students change the lengths of the sides of the triangle (a, b and c) by double-clicking on the values in the GSP document. They investigate each of the cases listed in the three tables (corresponding to three cases, elaborated below) provided by the teacher.

Sample table:

Length of side a / units	Length of side b / units	Length of side c / units	c^2 / units²	$(a^2 + b^2)$/ units²	Indicate the type of triangle with a tick.		
					Acute-angled	Obtuse-angled	Right-angled

[Note: Values of a, b and c have been omitted from the table in this article.]

This table is replicated three times for students to explore three cases:
Case 1: $c^2 > a^2 + b^2$
Case 2: $c^2 < a^2 + b^2$
Case 3: $c^2 = a^2 + b^2$

(2) Students observe the type of triangle formed (acute, obtuse or right-angled) by clicking on the "Form triangle" button.

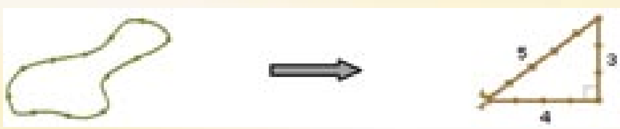
(3) After investigating each set of values (a, b and c) listed in the tables, students summarise their findings by answering the following questions:
a) If $c^2 > a^2 + b^2$, is the triangle acute, obtuse, or right-angled? __(obtuse-angled)_
b) If $c^2 < a^2 + b^2$, is the triangle acute, obtuse, or right-angled? __(acute-angled)__
c) If $c^2 = a^2 + b^2$, is the triangle acute, obtuse, or right-angled? __(right-angled)__

(4) Ask students if the converse of PT *likely* to be true or false.

[If you would like a copy of the GSP file, email us at shalom.safhaven@gmail.com.]

Although the syllabus does not require students to classify a triangle as acute-angled, obtuse-angled or right-angled given the lengths of its three sides, it might be useful to provide students with a holistic view of how the squares of the sides of a triangle are related to the type (acute, obtuse or right-angled) of triangle.

To conclude the activity, we can offer students a historical perspective of how ancient Egyptians used the converse of PT in architectural constructions, such as in the construction of the pyramids. To make a right-angled triangle, ancient Egyptians took a rope with 12 equally spaced knots in it, and then bent it to form a triangle with sides of lengths 3, 4 and 5. The angle formed by the sides of lengths 3 and 4 was used as a right angle.



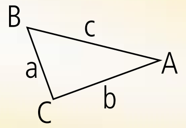
[Source: http://www.mcs.surrey.ac.uk/Personal/R.Knott/Pythag/pythag.html]

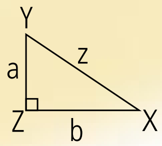
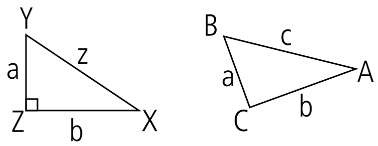
We conclude this article with an outline of a proof of the converse of PT.

5. A proof of the converse of Pythagoras' Theorem

Given: ΔABC with sides of lengths a, b and c and with $c^2 = a^2 + b^2$.

To prove: ΔABC is a right-angled triangle.



Outline of proof (prompts given to students)	*Diagrammatic and written form (worked out together with students)*
1. Draw a **right-angled** triangle XYZ with the two shorter sides having lengths *a* and *b* and hypotenuse of length *z*.	
2. Apply Pythagoras' theorem on triangle XYZ	$z^2 = a^2 + b^2$
3. Compare with given $c^2 = a^2 + b^2$ (Given)	$z^2 = a^2 + b^2$ and $c^2 = a^2 + b^2$ gives $z^2 = c^2$, so $z = c$
4. ΔABC and ΔXYZ are congruent (SSS) [See "Note" below.]	
5. Since ΔABC and ΔXYZ are congruent, then corresponding angles are equal.	$\angle C = \angle Z = 90°$ (proof complete)

[Note: It should be noted that at the Secondary 2 level, students would not have learnt the SSS test for congruency of triangles, which is used here in the proof of the converse of PT]

References

Knott, R. (2006). *The 3-4-5 triangle*. Retrieved April 20, 2007, from: http://www.mcs.surrey.ac.uk/Personal/R.Knott/Pythag/pythag.html

Leong, Y. H. (2007). Difficulties with geometry proofs. *Maths Buzz*, 8(1), (pp. 4-6). Singapore: Association of Mathematics Educators.

Ministry of Education. (2007). *Secondary mathematics syllabuses*. Singapore: Curriculum Planning and Development Division.

Serra, M. (2003). *Discovering geometry: An investigative approach*. Emeryville, CA: Key Curriculum Press.

Amazing Math Race 2007 in Junyuan Secondary School

Sharon Tan and Teng Sok Wah
Junyuan Secondary School

On 11th April 2007, the Mathematics department of Junyuan Secondary School organized the Amazing Mathematics Race for the second year running. Mrs Janet Oh, the Principal of Junyuan Secondary School, gave the opening address and flagged off the one hour race. Forty teams, comprising of 160 Primary Four to Six pupils from 22 East Zone schools competed in the event

Students from Junyuan Secondary School took charge of the planning and organizing the event, in consultation with their Mathematics teachers. The planning began two months ago with the underpinning philosophy that "learning Mathematics can be fun". The students were empowered to craft Mathematics activities and questions that were namely authentic in nature.

All stations were designed to engage the participants in the learning of mathematics, through hands-on activities and manipulatives. The stations focused on different mathematics topics. In one of the stations, where the theme was on circles, the activity designed required students to measure the circumference of the circle on a basketball court using a piece of string. The measurements calculated were then applied to find the circumference and area of the circle. At another station on speed, participants recorded the time taken to walk a distance of 5 metres to find the speed taken to walk that distance. With the calculations, the teams applied the values to the speed-time formula to find the distance covered in 1 minute. All designed activities involved the application of mathematics to real life problems such as the counting of tiles in the courtyard or finding the ratio of tables to benches in the canteen. In addition, teams also had the opportunity to create charts using Microsoft Excel in the computer laboratories.

Besides the display of mathematical competence by the competitors to apply and solve problems in the race, in designing the activities, Junyuan students also demonstrated their ability in making connections and relating mathematical concepts with everyday life. In addition to the task of crafting the questions, the students also came up with the marking scheme for the activities, communicated the objectives and instructions to the participants at the stations and demonstrated using manipulatives the mathematical concepts to participants who were lost in the midst of all the computation. In their quest to win the race, besides the mathematics knowledge, the participants also acquired good practices about teamwork and perseverance in the face of challenges as they attempted to complete the race within the stipulated time of an hour.

At the end of the race, fifteen teams walked home with trophies of bronze, silver and gold awards. Judging from the positive response of the participants, the students not only learnt the mathematics but most importantly, they were involved and had enjoyed inthe learning process.

The following comments were given by the participants when interviewed:

- *"It is enriching. It allows us to apply mathematics knowledge in the activities."*
  Jeremy Chee, Primary 6, MacPherson Primary School
- *"I learnt more Math in the race. I had fun in the race."*
  Lim Tse Puay, Primary 5, Bedok Green Primary School
- *"It is enriching. It gives us the chance to apply real life mathematics."*
  Agnesh Rai, Primary 6, MacPherson Primary School
- *"The race is enjoyable and very exciting. The Race lets us rack our brains and let us love Math more."*
  Aliah Bte Md Amin, Primary 5, Park View Primary School

Creating Impossible Figures When Changing Numbers in Geometry Problems

Helmer Aslaksen
National University of Singapore

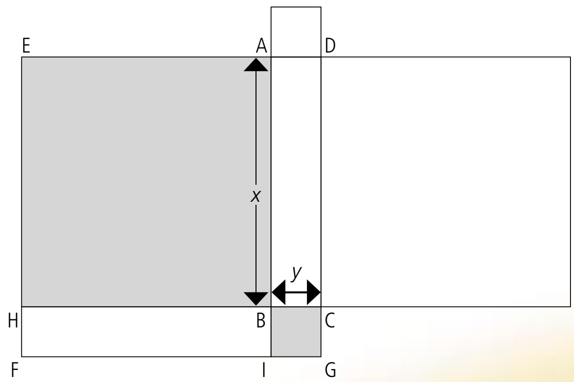
Many teachers try to create examination problems by changing the numbers in existing problems. In algebra problems, this is safe. (Ignoring the dubious educational value of trivial variations of standard problems) However, in geometrical problems this is likely to lead to impossible figures. The shape of the figures impose relations between lengths and areas, which are easily broken if the numbers are changed. It is essential to actually compute the lengths to see that the figures exist. Unfortunately, many teachers seem to have the mistake belief that it is sufficient to use "logical" arguments on non-existing figures!

There was an example of such an incorrect problem on the PSLE in 2005 [1]. In this note we will look at an another example from a test at a local school in 2007.

The questions says that the perimeter of the central rectangle ABCD is 20 units, that the sum of the areas of the four squares on the sides of ABCD is 80 units^2 and asks for the area of ABCD. The suggested solution implies that the length of DG, which is the sum of DC and CB, is 10 units. That means that the area of EFGD is 100 units^2. Since the sum of the areas the two squares EHBA and BIGC is 80/2 = 40 units^2, it follows that the area of ABCD equals (100–40)/2 = 30 units^2. Unfortunately, no such rectangle ABCD actually exists!

There are two simple ways to see this. The easiest is to observe that when we divide a square into two squares and two rectangles by dividing the sides in the same ratio (as with the square DEFG), then the sum of the areas of the two squares must be bigger than the sum of the areas of the two rectangles.

If we set AB = x and BC = y, then $(x-y)^2 = x^2+y^2 - 2xy \geq 0$, so $x^2+y^2 \geq 2xy$.



But in our "example", the claim is that the sum of the squares is 40, while the sum of the rectangles is 60!

We can also use algebra. We have
$2x+2y = 20$,
$2x^2+2y^2 = 80$.

Setting $y = 10-x$, the second equation becomes $x^2-10x+30 = 0$, which only has the complex solutions, $x=\frac{10\pm\sqrt{100-120}}{2}=5\pm\sqrt{-5}$

I hope this example will serve as a reminder to teachers to always check the validity of the figures when making changes to the dimensions. This can be done using algebra, trigonometry or dynamic geometry software like Geometer's Sketchpad.

References

Helmer Aslaksen, PSLE 2005 and Curry's Paradox, Mathematical Medley 32, (2005), no. 2, 2–9.

Some Issues on Teaching Mathematics Concepts at the A-Levels

Toh Tin-Lam
National Institute of Education

1. Introduction

The A-Level mathematics curriculum includes a higher level of abstract contents compared to the O-Level mathematics. However, many A-Level students may not be immediately ready to accept such abstract content. Consequently, teachers need to think of skilful ways to explain these concepts to their students. This makes the teaching of A-Level mathematics challenging.

In the process of attempting to explain the abstract concepts to their students, the teachers might end up over-simplifying abstract mathematical concepts that may either end up with developing a fixed mindset in their students or conveying wrong mathematical concepts to their students.

Many of these misconceptions are not "operational" in the sense that these misconceptions will not affect students' performance in the examinations. Unfortunately, these will pose as learning difficulties to the students for their further undergraduate studies in mathematics and other related fields. In this note, we will discuss some of such examples.

2. Sequence and Series

One main concept in the elementary undergraduate calculus concept involves sequence and series. There are several sections in the A-Level syllabus that deals with sequence and series: possibly, the students' first encounter of sequence occurs in the chapter *arithmetic and geometric progression*, infinite series under the chapter on *series and Σ-notation*, followed by *binomial* and *power series.*

At A-Levels, a sequence is recognized as a (finite or infinite) set of numbers separated by commas; while the sum of this set of numbers is a series.

2.1 Concepts involving Infinite Series

In the A-Levels lecture notes, the concept of an infinite series is usually presented in some form as:

*The sum to infinity of a converging sequence is the addition of infinite number of terms of a sequence.**

In real life, however, it is *impossible* to find the sum of infinitely many numbers, as nobody would do addition of numbers indefinitely! Understanding infinite sums in this way, students may find the concepts of infinite series "meaningless" (Toh, 2006). The concept of the sum of an infinite series should be properly understood from the limit of a sequence of partial sums, rather than the infinite addition of numbers.

As an illustration: we shall examine what $\sum_{r=1}^{\infty}\frac{1}{2^r}$ means. Let $S_1=\frac{1}{2}$, $S_2=\frac{1}{2}+\frac{1}{2^2}$,……$S_n=\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots=\frac{1}{2^n}$ By evaluating the sums $S_1, S_2, S_3, \ldots\ldots S_n$, one realizes that the sequence S_n tends arbitrarily close to the number 1. Thus $\lim_{n\to\infty} S_n=1$, The sequence $\lim_{n\to\infty} S_n$ is denoted by $\sum_{r=1}^{\infty}\frac{1}{2^r}$. This is the actual meaning of the sum of an infinite series (if it exists).

In summary from the discussion from the above paragraphs: when properly expounded, infinite series should be understood from the concept of limits in calculus, and calculus is generally difficult for A-Level students. On the other hand, oversimplifying the concept of infinite series as adding infinitely many numbers may render the concept meaningless to students.

2.2 Sequence and Series

Because some basic knowledge of infinite series is required for the A-Level curriculum, teachers would dedicate some time to teach their students basic concepts on limits of sequence before discussing the sum of infinite series. Thus, teachers need to link up the concepts of limits of sequence and the sum of infinite series rather well and, more fundamentally, to have a sound understanding of the fundamental calculus.

We have discussed in the previous section that an infinite series can be seen as a sequence of partial sums of the sequence. There are two concepts involved here: the limit of a sequence and the limit of the series. If the limit of a sequence exists, then the sequence is said to be converging; if the limit of the series exists, then the series is said to be converging.

For illustrations:
Consider the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots, \frac{n}{n+1}$. The sequence is converging to the value 1. We write $\lim_{n\to\infty}\frac{n}{n+1}=1$. However, the corresponding sequence of partial sums $\sum_{r=1}^{n}\frac{r}{r+1}$ does not converge to any value, hence we say that the infinite sum $\sum_{r=1}^{\infty}\frac{r}{r+1}$ does not exist.

Thus, it is possible that a sequence converges but the infinite series from the limit of the corresponding sequence of partial sum does not exist.

However, it is a well-known theorem from calculus that if the infinite series $\sum_{n=0}^{\infty} a_n$ (i.e. the limit of the sequence of partial sum, or $\lim_{n\to\infty}\sum_{r=1}^{n} a_r$) exists, then $\lim_{n\to\infty} a_n$ exists and must be equal to zero. In short, limits of sequence and infinite series are different and yet tightly related concepts; these need to be addressed carefully in the course if limit concepts are taught in the curriculum.

A-Level syllabus does not require the students to know in depth about these distinctions; students are just required to know how to check the convergence of some special series and find the infinite sum if they exist. As a result, even when these confusion are not properly addressed, students' performance in the examination will not be too adversely affected. However, the misconception will remain with the students and may affect them for their subsequent learning of higher level mathematics.

2.3 Some tutorial questions on Sequence and Series

This section discusses some questions set in tutorial for students on sequence and series. As a first example, consider the following question for our discussion.

Question: Find the *n*th term, u_n, of the following sequences.

(i)	5, 7, 9, 11,…….	(iv)	$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots\ldots$
(ii)	-6, 8, -10, 12,……..	(v)	$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots\ldots$
(iii)	3, 5, 9, 17,…….	(vi)	0, 1, 5, 23,………

This question serves as a good recap to what pupils have learnt in their secondary school days on finding a "formula" using algebraic expression. It is an important preliminary skill for students to write a sequence using the sigma notation.

However, it should also be realized that ***the formula that one has generalized is NOT THE ONLY choice for forming the sequence***. In fact, one can write down any arbitrary sequence and then try to derive a formula for that sequence, for example, by using Lagrange interpolation polynomial. An interesting classic example was discussed in Section 5 of Toh (2006): the sequence whose first few terms are 1, 2, 4, 8, 16,……., might be suggestive of a rather simple formula $u_n=2^{n-1}$; however, it is part of a series of a more complicated formula, see Pg 38 Toh (2006).

Thus, the important part of this part of writing formula is not to test how good a student is in recognizing the formula by any clever means; rather it is more important that a student can write out a possible general formula for u_n and later use the Σ-notation to express a series in a succinct way. Teachers should bear in mind that the entire objective of the above activity is NOT to get pupils to formulate the general term by "clever" means but to have a sound understanding of the use of sigma notation.

It is also important that students should not be asked, as far as possible, to be engaged in meaningless, or even mathematically wrong, computations. Here we shall use another tutorial question part for illustration.

1. Write out explicitly the following series
   (a) $\sum_{r=1}^{\infty}(2r^3-1)$

2. Write the series in Σ-notation:
   (a) $2-4+6-8+10-12\ldots\ldots=$

The series in Questions 1(a) and 2(a) are not convergent, thus the sum $\sum_{r=1}^{\infty}(2r^3-1)$ is meaningless. This will further reinforce the wrong concept of infinite series as adding infinitely many numbers.

3. Concepts related to Calculus

Here we shall further discuss two specific examples related to Calculus.

3.1 Finding asymptotes of hyperbola

Sometimes incorrect mathematical reasoning could lead to the correct answer; this could lead to further misconception, especially when the students start learning other topics. One example is about finding the asymptotes of the hyperbola with equation given by $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$. Here we shall present two explanations:

Explanation 1 Consider $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$. Let x and y tend to (positive or negative) infinity, then the left hand side of the equation becomes infinity minus infinity, *which is zero*. Thus, the asymptote can be obtained by $\frac{y^2}{a^2}-\frac{x^2}{b^2}=0$ and further simplifying it leads to $y=\pm\frac{a}{b}x$, which are the asymptotes of the hyperbola.

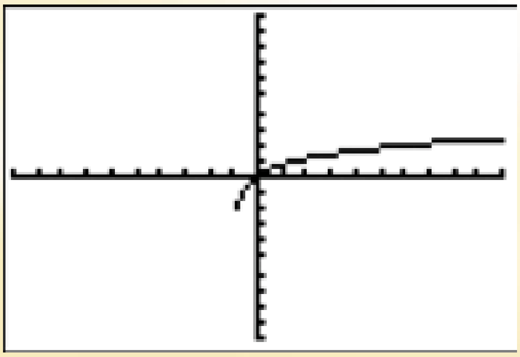
Explanation 2 Consider $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$. By rearrangement, the equation becomes $\frac{y^2}{a^2}=\frac{x^2}{b^2}+1$. When x and y are extremely large in magnitude, the terms $\frac{y^2}{a^2}$ and $\frac{x^2}{b^2}$ are both extremely large. Since these two extremely large terms differ by only 1, we can say that $\frac{y^2}{a^2}\approx\frac{x^2}{b^2}$. Thus, $\frac{y^2}{a^2}=\frac{x^2}{b^2}$, which reduces to $y=\pm\frac{a}{b}x$, gives the equations of the two asymptotes of the hyperbola.

Notice that both Explanation 1 and Explanation 2 give the correct asymptotes to the hyperbola; however, Explanation 1 leads to the wrong calculus concept that "infinity minus infinity equals zero".

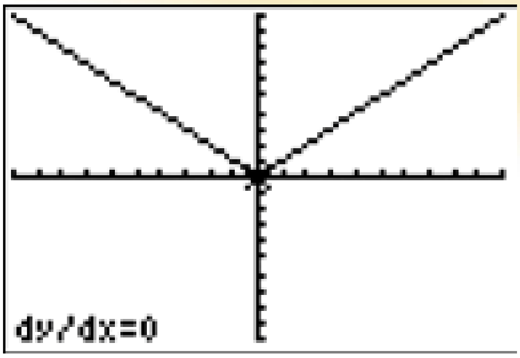
3.2 Use of Graphing Calculator

Graphing calculator has been introduced into the A-Level mathematics curriculum to allow students to explore mathematical concepts. It encourages the students be more independent learners. However, students also need to be cautious when using the tool for independent study.

For example, a careless student without knowledge of calculus may take the shape of the logarithmic function $y=\ln(x+1)$ directly from the calculator as one that looks like below.



The concept of asymptotic behavior, due to the resolution of the calculator, is not easily demonstrated without a sound understanding of the mathematical concepts. Teachers.



Other misconception involves the use of graphing calculator to handle non-differentiable points. For example, in trying to find the value of $^{dy}/_{dx}$ of the function $y=|x|$, the calculator gives the value of zero as shown below.

In A-Levels, rarely will students be asked to handle non-differentiable points. However, such tools for independent learning might still lead to some misconceptions which would surface only at undergraduate level.

4. Discussion and Conclusion

It is impossible to lead A-Level students to go through the entire rigors of the undergraduate mathematics; however, one can look for creative ways to explain or demonstrate mathematical concepts to students, see for example Toh (2002). Although the rigors are dispensed with, mathematical misconceptions can also be avoided while offering students another perspective of looking at the mathematics problems instead of rote learning.

While it is important to help our students to excel in the A-Level examinations, it would be better if teachers could take precautionary measure to minimize the arising of misconceptions in mathematics as far as possible.

References

Singapore Ministry of Education, 2006 A-Level Mathematics syllabuses, Curriculum Planning and Development Division, 2004.

Toh T.L. On Teaching Binomial Series: More Meaning and Less Drill, International Journal of Mathematics Education in Science and Technology, Vol 34(1), P115 – 121, 2002.

Toh T.L. Mathematical Reasoning from O-Level to A-Level, Mathematical Medley, Vol 33(2), P34 – 40, 2006.

Design, Colour Separation & Print by Oxford Graphic Printers Pte Ltd