ASSOCIATION OF MATHEMATICS EDUCATORS
Website: http://math.nie.edu.sg/ame/

Maths Buzz

Vol.10 Issue 1 | MICA (P) No. 277/09/2008 | October 2008

Contents

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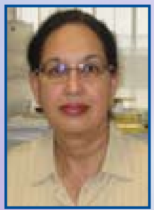
President's Message



Dear AME Members,

The Association of Mathematics Educators is now 14 years old. Under the able leadership of the past 7 executive committees the Association has gained not only national but also international recognition. Our publications, Maths Buzz and The Mathematics Educator have both served their purposes well. The Maths Buzz has helped to wet the appetite of our members – kept them posted briefly about upcoming events and shared with them some interesting snippets on mathematical ideas or teaching episodes. The Mathematics Educator has been very successful in publishing research papers related to the teaching and learning of mathematics. It is currently a much sought after publication by international researchers. This is certainly a significant milestone for the Association. To all our past and present editors of Maths Buzz and The Mathematics Educator we say, Thank You and Well Done!

In line with its aims and goals, the Association has to date provided numerous professional development and enrichment activities for members and mathematics teachers. It has organized numerous workshops by local as well as international experts. It has also co-organized both national and international conferences such as the ERA-AME-AMIC Conference in 1999 with Educational Research Association of Singapore and ICMI - EARCOME in 2002 with Mathematics & Mathematics Education AG at NIE. The contributions of the 6th and 7th executive committees in making an annual Mathematics Teachers Conference as part of the activities of the Association are laudable and deserve commendation! To date, if you may recall four Mathematics Teachers Conferences have been held:

Date	Theme
2nd June 2005	Assessment
1st June 2006	Enhancing Mathematical Reasoning
1st June 2007	Mathematical Literacy
29th May 2008	Mathematical Problem Solving

All four conferences were very well attended and received by members of the Association and mathematics teachers.

At the 15th annual general meeting of the Association, I was elected President of the eighth executive committee of the Association. This is my fourth time as President of the Association. The eighth executive committee of the Association will strive to do their best for the Association. We are already planning the next Mathematics Teachers Conference to be held on 4th June 2009 with the theme: Mathematical Applications and Modeling. As we work towards the aims and goals of the Association we need your support and cooperation for the continued well-being of the Association and to scale it to greater heights. We value your views and feedback. Please feel free to e-mail us about any matter related to the Association. The Association's homepage is http://math.nie.edu.sg/ame/.

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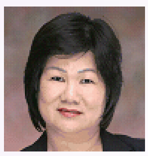
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Association of Mathematics Educators

On relationships between Volume & Surface Area, Area & Perimeter: Arousing students' curiosity in the World of Mathematics

Chen Weiqiang, Toh Tin Lam
National Institute of Education

Problem solving is the core of the Singapore mathematics Curriculum. A first step in making problem solving meaningful is perhaps arousing the curiosity in the subject itself. In this note, we shall illustrate with examples.

Students first learnt about the formulae for volume (V) and surface area (A) of sphere as $V=\frac{4}{3}\pi r^3$ and $A=4\pi r^2$ in Lower Secondary. Few textbooks would elaborate further on how these formulae could be derived. At most, the textbooks would tell students that the formulae could be derived by Calculus, which they would learn in Upper Secondary.

However, if one were to explore into the history of the development of the concepts, one would have a enlightened view of how the ancients dealt with these formulae, which could excite one into the learning of Mathematics. We strongly encourage readers into exploring more into the history of Mathematics to see how the various mathematical concepts were grappled.

After a students has learnt Calculus, he may observe that the surface area A of a sphere is the derivative of the volume V with respect to its radius r, i.e. if $V=\frac{4}{3}\pi r^3$, and $\frac{dV}{dr}=4\pi r^2=A$. Is this a mere coincidence? Or are there more to it? Unfortunately, many secondary textbooks do not mention much.

However, by using the definition of the derivative, it is not difficult to see that $\frac{dV}{dr}=A$ for a sphere is NOT a mere coincidence. See Figure 1 below.

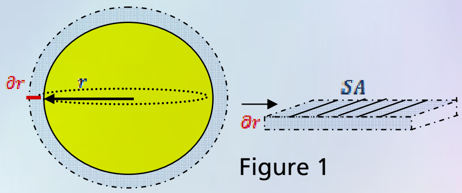

Figure 1

Generalisation

At first glance, the relationship $\frac{dV}{dr}=A$ seems to hold for the case of the sphere but not for other shapes. For example, differentiating the volume of a cube (with length of its side l) with respect to its length gives: $\frac{dV}{dl}=3l^2$ which differs from the surface area of the cube 6 l^2. However, we will illustrate that generally such a relationship can be established for other solids when we attempt to formulate the relationship in the setting of infinitesimals and define ∂r in equation differently.

It turns out that the required formulation is $\partial V \approx SA\ \partial r'$ (1)
where ∂V denotes the change of volume of the object that would result from coating it with a uniform layer of thickness $\partial r'$ and SA represents its surface area. We show this formulation is satisfied in the particular cases when the solid is a sphere, a cube and a cylinder.

Note that in the case of the sphere, Equation (1) is easily satisfied by setting $\partial r'=\partial r$ and SA=$4\pi r^2$.

Let us now consider the case of the cube. If we ignore the corners of the cube, we observe that by extending the length of each edge from each corresponding face of the cube as shown in Figure 2, we have $\partial V=6\cdot(l^2\partial r')=SA\ \partial r'$

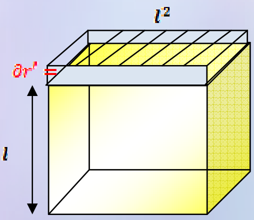

Figure 2

We consider the case of the cylinder as the next example. In this case, we now only get an approximation as stated in equation (1) instead of equality as shown in the previous two examples. Recall that for a cylinder of base radius r and height h, its volume and surface area are given by the following formulae:

$$V=\pi r^2 h \qquad SA=2\pi r^2+2\pi rh.$$

Now let us refer to Figure 3 and calculate the corresponding change of volume ∂V as shown in Figure 3.

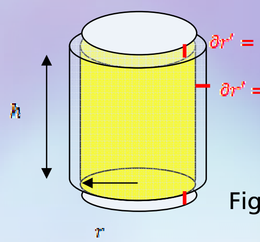

Figure 3

Consider ∂V_1 to be the corresponding vertical volume change at the two ends of the circular faces. Then

$$\partial V_1=2\cdot\pi r^2(h+\partial r')-2\cdot\pi r^2 h$$
$$=2\cdot\pi r^2\partial r'.$$

Let ∂V_2 to be the corresponding horizontal volume change of the cylinder. We have

$$\partial V_2=\pi(r+\partial r')^2 h-\pi r^2 h$$
$$=2\pi rh\partial r'+\pi h(\partial r')^2$$
$$\approx 2\pi rh\partial r'$$

because $\partial r'$ is small. Thus the total corresponding volume change of the cylinder is

$$\partial V=\partial V_1+\partial V_{20}$$
$$\approx(2\pi r^2+2\pi rh)\,\partial r'$$
$$=SA\ \partial r',$$

which is what we wanted to show.

In fact, the relationship in Equation (1) is also true for solids like cones and pyramids.

Relationship between Area and Perimeter of Plane Figures

At the end of this article, we like to point out that an analogous relationship generally exists between the area of a figure and the perimeter of a two-dimensional figure. The relationship can be stated formally as:

$$\partial A \approx P\ \partial r' \qquad (2)$$

where ∂A denotes the change of area of the figure that would result from extending each differentiable point on the circumference of the figure by uniform length ∂r and P represents its perimeter. We will just show that equation (2) is satisfied particularly when the figure is a circle.

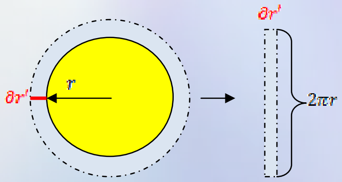

Figure 4

Take $\partial r' = \partial r$, then it is easy to see that

$$\frac{dA}{dr} = 2\pi r = P$$

and thus $\partial A = P\,\partial r'$. A comparison could be made between Figures 1 and 4 to reveal an analogous relationship. Likewise, the reader could show that such a relation also holds true in the case of a square. The main ideas required are somewhat similar and interested teachers might explore more into these results and might even work them out as exercises or excite their students into doing Mathematics projects to extend these results further.

Conclusion

There are many such interesting results which are not handled sufficiently in the school textbooks, most likely these material are not in the examination syllabus. However, using this as an opportunity to arouse students' curiosity might make students more excited about Mathematics, and will definitely help in their mathematical problem solving.

The Historical Development of Mathematics: A Pedagogical Resource

Jaguthsing Dindyal

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"... the easiest and fruitful way of presenting a large part of mathematics is either against a background of history or in the context of a historically-based philosophy of mathematics" (Rogers, 1983, p. 401).

Mathematics has had a long and rich development spread over several millennia. However, the polished presentations of mathematics in school textbooks and in the mathematics classroom somehow define away this past. Mathematical concepts and ideas that took several centuries to develop are now presented to students within a few lessons. Kline (1972) claimed that the courses fail to show the struggles of the creative process, the frustrations, and the long arduous road mathematicians must travel to attain a sizeable structure. He added that once students become aware of the historical development of the subject, they will not only gain insight but derive courage to pursue tenaciously their own problems and not be dismayed by the incompleteness or deficiencies in their own work.

How can the history of mathematics be used in the classroom? The issue is not about teaching the history of mathematics per se but using the historical development of the subject as a pedagogical tool. Jones (1989) cautioned that the history of mathematics will not function as a teaching tool unless the users (1) see significant purposes to be achieved by its introduction, and (2) plan thoughtfully for its use to achieve these purposes. The age and background of the students and the "ingenuity" of the teacher are certainly important factors in determining the approach to be used. There are several ways in which the history of mathematics can be incorporated in school mathematics and in the classroom. For example, if the teacher wishes to focus on major developments in a chronological order then the approach will have to be different from one who focuses on major developments in specific geographical regions. Another teacher may wish to focus on solving problems as they might have been solved in the past. There can be no single cookbook approach. Some ideas that may be considered include:

1. The Development of Mathematical Concepts and Ideas

Mathematical concepts have developed over a very long period of time. For example, if one considers the concept of number then it would be worthwhile to know how the Hindu-Arabic numeration system developed over time. Why is it a better system compared to other systems? How and when did we come to use negative numbers and complex numbers? One can also look into the development of individual branches of mathematics. For example, how did calculus develop into such an important branch of mathematics? While calculus developed on shaky foundations, how was it put on a firm footing? How did geometry change over time? How did algebra develop as a branch of mathematics? If a chronological approach is adopted, then teachers can focus of the major developments in mathematics at particular checkpoints in recorded history all around the world.

2. The Contributions of Great Mathematicians

Some mathematicians have made major contributions to mathematics. Archimedes, Euclid, Descartes, Fermat, Newton, Euler, and Gauss are just a few of the eminent persons whose inspiring lives and works would certainly be of interest to the new generation of students. The teacher can focus how the mathematicians created or invented new mathematics and solved difficult problems using very simple tools. The lives and works of some prominent women in mathematics can be highlighted to inspire more girls to do well in mathematics. For example, Hypatia of Alexandria (~370 – 415 AD) can be considered as one of the first woman with a deep interest in mathematics and Sofia Kovalevskaya (1850 -1891) and Emmy Noether (1882 – 1935), in more recent years can be quoted as well.

3. The Contributions of Some of the Great Civilizations

In all civilizations mathematics has played an important role. Recorded history takes us far back to the Babylonians and Egyptians for whom mathematics was an empirical science and its knowledge was shared by a select group of persons. Mathematics was a "tool in the form of disconnected simple rules" (Kline, 1972, p. 22). It was the Greeks who organized mathematics into a coherent whole and gave it a very rigorous foundation. They linked mathematics, which was mostly geometry for them, with the highest form of reasoning and hence it formed part of their philosophical debates. Although the mathematics we teach is schools is strongly Eurocentric, we should not forget to celebrate the mathematics from other civilizations such as the Arabs, Chinese and the Indians. This will add a human touch to a subject that is usually taught devoid of any historical roots.

4. Misconceptions and Errors

Misconceptions and errors have always been part of the historical development of mathematics. Famous mathematicians have held very erroneous views about certain aspects of mathematics. For example, the adherents of the Pythagorean school believed that a line is made up of a finite number of points and could not accept the concept of incommensurables which involved irrational numbers. The Greeks failed to comprehend the infinitely large, the infinitely small, and infinite processes. Aristotle has been quoted as saying that the infinite is imperfect, unfinished and therefore unthinkable; it is formless and confused (see Kline, 1972, p. 175). On the other hand D'Alembert advanced physical, analytical, and geometrical arguments that log(-1) = 0. (Kline, 1972, p. 421). It is claimed that Euler thought of infinity as a number and that 0/0 is any number since n.0 = 0, hence n = 0/0. The use of examples that illustrate some misconceptions from the past can help students understand that everybody can make mistakes and learning from our mistakes is a normal process.

5. The Development of Mathematical Language and Symbolism and other Mathematical Tools

The deductive nature of mathematics is certainly a Greek legacy. With the advent of Algebra, the language of mathematics has changed significantly over time. There is a higher dependence on symbolic language in any form of mathematics. The mathematical language has become highly technical as well. Many mathematical concepts have conventionally agreed symbols which impart a universal aspect to these concepts. How these symbols have developed over time is an interesting matter for discussion in class. There have also been various types of calculating, measuring and drawing tools which have somehow influenced the mathematical landscape. The evolution of the mathematical language can be in itself a matter for discussion in class.

6. Mathematical Problems Solved and Unsolved, Paradoxes

At different points in time, mathematical problems have been solved differently. The Greeks reduced all problems to geometry. Following the advent of algebra, the algebraic approach became a popular approach for solving problems. Some problems have taken a long time to be solved. The solution of Fermat's Last Theorem was a challenge for a long time before Andrew Wiles solved it. The four-colour problem also took a while before it was solved using computers. We still have many problems which have not been solved. Famous among these is the Goldbach conjecture. Such problems can be highlighted for students who can then be challenged to look for solutions. While we do not expect students to come up with easy solutions, we do expect students to understand that much of mathematics is created or invented when we spend time on such problems. Also, paradoxes have played an important role in advancing mathematical knowledge. Paradoxes such as Zeno's, Burali-Forti's or Richard's can also be used to challenge students to think deeply about some seemingly simple ideas.

To conclude, it can be said that there are many ways in which the teacher can use the history of mathematics in teaching. The teacher need not be an expert in the history of mathematics. Now, there exist many resources about the history of mathematics such as pictures, films, videos, slides and so on, that makes the task of the teacher easier. Many ready-to-use resources are available online for the teacher, as well. The history of mathematics is a worthwhile avenue to explore for helping students to learn mathematics.

"The roots of the present lie deep in the past and almost nothing in that past is irrelevant to the man who seeks to understand how the present came to be what it is" (Kline, 1972, p. viii).

Some online resources:

1. St Andrews MacTutor History of mathematics Site: http://www-groups.dcs.st-and.ac.uk/~history/
2. Chinese number system: http://www.mandarintools.com/numbers.html
3. General topics:
   a. http://forum.swarthmore.edu/~steve/steve/mathhistory.htm
   b. http://wwww.maths.tcd.ie/pub/HistMath/HistMath.html
   c. http://www.dcs.warwick.ac.uk/bshm/resources.html
4. Uses of mathematical symbols: http://members.aol.com/jeff570/mathsym.html
5. Use of words in mathematics: http://members.aol.com/jeff570/mathword.html
6. International Journal for the History of Mathematics Education: http://www.tc.columbia.edu/centers/ijhmt/index.asp?Id=Journal+Home
7. HPM is the International Study Group on the Relations between History and Pedagogy of Mathematics affiliated to the International Commission on Mathematical Instruction (ICMI). http://www.clab.edc.uoc.gr/hpm/about%20HPM.htm

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A Special Rule – a Good Problem For Upper Primary School Pupils and Teachers

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Abstract

Many students have a fear of 'proofs' in mathematics. We hope that a positive attitude could be instilled from upper primary school students when they are in the stage of developing skills in logical or deductive thinking.

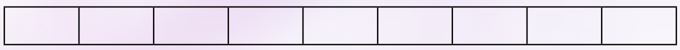
The problem on constructing a special rule is given to an upper primary school pupil. In the process of solving the problem the child goes through the several stages of problem-solving namely, trial and error, discovery, cases, conjecture and conclusion. Although, the child may not have the ability to write down a 'proof' in this early stage, he is proud to have 'proven' his conclusion.

The special rule problem can also be given to trainee teachers or teachers during a workshop. What could a better way for one to appreciate the process of solving mathematical problems than to actually go through the process!

Lastly, we look at possible problems generated from the special rule problem.

The problem

My son was given the following problem in his class:
You are given a 15cm-ruler.



How would you place 4 markings on your special rule so that you can measure 1cm, 2 cm, 3 cm, ... , 14 cm,15 cm?

Let us explain the meaning of 'measure'.
"Being able to measure a length of 5 cm" means that on the special rule, you can have two markings enclosing exactly a length of 5 cm.

For an example,

Having 4 markings at position 4cm, 7 cm, 9cm and 10cm from the left, this special rule can measure the following:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15.

However, the following measurements are not possible:
12, 13, 14.

Note that although you can have 12 = 10+2, you don't have two markings on this special rule that give a length of 12 cm.

Discovery process via Trial and Error

By trail and error, my son worked on various markings aiming to get a 4-marking rule with the desired property. He started with having his special rule being able to measure 1 cm, 2 cm, 3 cm, …

During the process, he has noted that by having a marking at 2cm (say), he could measure 13 cm (from 15-2) using the right end of the rule. He has also noted that after having a marking at 2cm, the next marking at 5 cm (say) will provide the following additional measurements: 3cm, 5cm, 10cm.

After several attempts, he came up with the following major observation:

There must be a marking at 1cm from the left in order to measure 1 cm and 14 cm.

With this, he has also noted that

There must be a marking at 13 cm from the left to allow the measurement of 2 cm and 13 cm.

By trial and error, he discovered that

Conjecture: Any 2 additional markings that he made should not give the same measurement that was already obtained by the previous markings.

With some arguments (which we omitted here), he concluded proudly that he has proven the following:

Conclusion: There is no such 4-marking special rule.

In the next section, we shall discuss rigorous arguments leading to his conjecture and his conclusion.

More rigorous argument.

As observed, the 4-marking special rule must satisfy the following properties:

Property 1. There must be a marking at 1cm from the left in order to measure 1 cm and 14 cm.

Property 2. There must be a marking at 13 cm from the left to allow measurement of 2 cm and 13 cm.

Next, we looked at the maximum number of different additional measurements that could be produced with each additional marking, and we tabulated here:

Marking	Maximum number of different additional measurements
0	1 (to measure 15cm)
1st	2
2nd	3
3rd	4
4th	5

For example, after placing the second marking, one can check that the third marking can produce at most 4 different additional markings.

From what we have tabulated, the total number of different measurements obtained with 4 markings is at most 15, from the sum (1+2+3+4+5).

Now, we shall justify his conjecture.

If a particular marking produces at least one measurement that is already obtained, then the total number of different measurements obtained for the 4-markings rule would be at most 14. Therefore, there is at least one measurement that will be excluded.

The above argument leads us to state the next property for the special rule.

Property 3 If we were to have a 4-marking rule with the desired property measuring each measurement from 1 cm to 15 cm, then each additional marking made should produce measurements not obtained in previous markings.

To argue that it is impossible to have the special 4-marking rule, we proceed to `construct' our rule based on the three properties.

For the 15 cm rule, we must have two markings at the 1 cm and 13 cm from the left. This gives us the following measurements:
1, 2, 12, 13, 14, 15.

In order to measure 11 cm, the 3rd marking can be placed at 2cm, 4cm, 11cm or 12 cm. However, bearing in mind that we should produce 4 different additional markings, we find that the third marking can only be placed at 4cm from the left. (Alternatively, this can be checked easily by considering all possible placements for the 3rd marking, refer to Table A in Appendix.)

With the 3rd marking at 4 cm from the left, we have the following four additional measurements:
3, 4, 9, 11

Now, if we can place the 4th marking such that it gives only new measurements, then we would have all 15 measurements, and hence have constructed a desired special rule. Let us place the 4th marking to avoid having a repeated measurement.

It is easy to see that placing the 4th marking between 1 cm to 4 cm, or 13 cm to 15 cm, (from the left) will provide at least one repeated measurements.

Since the previous three markings allow measurements from 1 cm to 4cm, we note that having the 4th marking between 5cm to 8cm will give a repeated measurement. The same argument applies to placing the 4th marking between 9cm to 13 cm.

Therefore, there is no position where the 4th marking could be placed to produce additional five different markings.

And, we announce the following result:

There is no 4-marking rule, of length 15 cm, that provides measurements from 1 cm to 15cm.

Conclusion -- extensions and generalization

One of our purposes of writing this article is to illustrate that an upper primary school pupil has the ability to put forth some rigorous arguments which are required in mathematics or sciences. Pupils at this age should be encouraged to develop such thinking skill leading towards a positive attitude towards rigor in arguments.

This above problem is considered a good problem as a child has to go through the process of trail and error to discover some properties, to argue various possibilities by considering cases keeping track of observed properties that must be fulfilled, and finally to realize that "YES, I CAN PROVE IT!"

This problem can also be given to trainee teachers or teachers during a workshop. Trainee teachers or teachers who have worked on the problem will be able to appreciate the various methods in solving a mathematical problem. Moreover, the instructors can guide them towards writing down (mathematical) statement made and work towards 'writing a proof '. It is known that many students are afraid of proof-problems. However, if teachers have a positive attitude towards a proof, so will most of their students!

Lastly, the above problem generates other interesting problems which we will only list some of them.

1. Is there a pattern in the maximum number of different additional measurements made by each new marking?

   The table of maximum number of different additional measurements provides the keen eyes to observe that there is a pattern on the number made. If there are n markings, then the (n+1)th marking will provide at most (n+2) different and additional markings.

2. Is there a pattern in the maximum total number of different additional measurements made by n markings?

   The table also indicates that the maximum of the total number of different measurements made by n markings is (1+2+3+ … +n+(n+1)) = (n+1)(n+2)/2.

3. What is the minimum number of markings to be made on a 15cm rule ? Are there more than one way of placing such markings?

4. Is there a special rule of other length? For example, is there a special rule of length 10 cm and with 3-mmarkings that can measure 1 cm, 2 cm, up to 10 cm?

   By looking at the total number of different additional measurements made by some number of markings, the child will note that

   For a 3cm-rule, it is possible to have a 1-marking special rule to measure all measurements from 1 cm to 2 cm. Moreover, this is the least number of marking one can have.

   For a 6cm-rule, it is possible to have a 2-marking special rule to measure all measurements from 1 cm to 6 cm. As above, this is the least number of marking one can have.

   However, for the 10 cm-rule, it is not possible to have 3-marking special rule to measure all measurements from 1 cm to 10cm.

Appendix:

Table A

Position of 3rd marking (from the left)	New additional measurements
2	1, 2, 11, 13 (1,2 & 13 are not new measurements)
3	2, 3, 10, v12 (2 is not a new measurement)
4	3, 4, 9, 11
5	4, 5, 8, 10,
6	5, 6, 7, 9
7	6, 7, 6 , 8 (Rejected since 6 is repeated)
8	7, 8, 5, 7 (Rejected since 7 is repeated)
9	8, 9, 4, 6
10	9, 10, 3, 5
11	10, 11, 2, 3 (2 is not a new measurement)
12	11, 12, 1, 2 (1 & 2 are not new measurement)

Geometry Proofs

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Since the introduction of "Proofs in Geometry" in the 2007 revised Additional Mathematics syllabus, there has been increased discussion among mathematics teachers on how to help students grasp the concepts and techniques of geometry proofs. In my earlier article in this magazine (Leong, 2007), I mused about some hypothetical difficulties associated with students' learning of proofs. Since then, a number of teachers have indicated that those difficulties I mentioned in the article were indeed experienced by some students. In addition, they pointed out other difficulties they faced in teaching geometry proofs.

It is acknowledged that for students to gain proficiency with proofs, they need to become familiar with a number of tools such as the idea of proof, the language of warrants, and the symbols for labeling geometrical objects. The learning of these tools for proofs takes time. However, most teachers find that "Geometry Proofs" within the Scheme of Work (SOW) are allocated at most three weeks of instructional time. One major struggle that teachers face is the time pressure how to help students gain familiarity of proofs within the limited time?

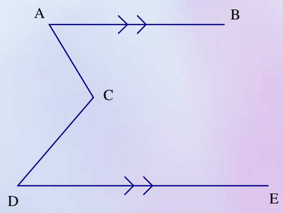
I make an admittedly bold proposal: Since students need time to understand the idea of proof and the SOW at Upper Secondary Additional Mathematics cannot provide sufficient time, why not start teaching proofs earlier? Why not start even at the Lower Secondary? I can almost hear an immediate retort: If students find it hard at Upper Secondary, how do you expect students at Lower Secondary to cope? For the rest of this article, I shall outline my proposal of how proofs can be introduced into Lower Secondary mathematics as a bridge to Upper Secondary proofs while keeping the contents generally manageable to students.

From "Problem to Solve" to "Problem to Prove" in Lower Secondary

One of the first challenges to overcome is students' unfamiliarity with the concept of "proof". At the Lower Secondary levels, most students are only familiar with "problems to solve" rather than "problems to prove"[1]. For this reason, I recommend that we *do not* begin the teaching of proofs by using "problems to prove"; rather, we use "problems to solve" as a bridge to lead students to proofs. Consider the Problem 1b below as an example of a "problem to prove".

Problem 1b:

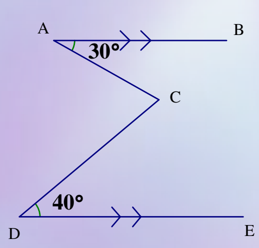
As shown in the diagram on the right, AB and DE are parallel line segments. Given that C is a point that lies between AB and DE, show that ∠ACD = ∠BAC + ∠EDC.



While having Problem 1b in mind, our classroom instruction can begin instead with a similar "problem to solve", such as Problem 1a:

Problem 1a:

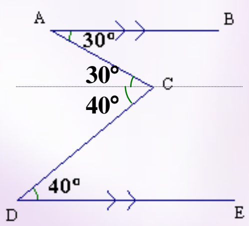
As shown in the diagram on the right, AB and DE are parallel line segments. Given that that C is a point that lies between AB and DE, and that ∠BAC = 30 and ∠EDC = 40, find ∠ACD.



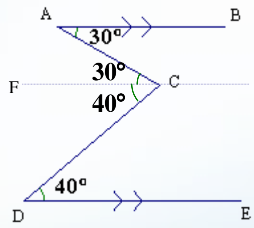
Notice the similarities between Problem 1a and Problem 1b. The former is presented with numeric quantities in the conditions so that it will appear less daunting for students as a first-step towards proof. The processes in the solution of both problems are similar too (and intentionally so). In the next section, I will present an approach to solving Problem 1a. However, the purpose is not in *solving* the problem per se; rather, the process is constructed with a view of leading students towards proving —as in Problem 1b.

Solving a problem with a view towards proof

First present the "(1) Given (condition)" and the "(2) Find (conclusion)". The purpose is to make explicit the condition and the conclusion in the problem. This is an important disposition subsequently in the prove-mentality. Next, provide a (3) diagrammatic overview of the solution strategy. The table below is an illustration of the steps involved up to this stage of the solution presentation.

(1) Given:	AB // DE ∠BAC = 30˚ ∠EDC = 40˚	(3) Diagrams
		
(2) Find:	∠ACD = _______	

Under conventional circumstances, students (and maybe teachers) will be quite happy to stop at this juncture because they would have obtained the answer. However, recall that the main reason for introducing Problem 1a is not primarily about getting the answer; rather, it is meant as a bridge to lead students towards proof (Problem 1b). For this purpose, the focus of the solution presentation enters at this stage into a critical juncture. Teachers can at this point emphasise the shift of focus from "answers" to "(4) working". In other words, the challenge is to translate the diagrammatic strategy into a textual form of communication. The final step (5) is to revisit the working by inserting warrants at suitable junctures. The completed presentation is illustrated in the table below.

(1) Given:	AB // DE ∠BAC = 30˚ ∠EDC = 40˚	(3) Diagrams
(4) Working:	∠ACF = 30˚ (Alt. angles, // lines) ∠DCF = 40˚ (Alt. angles, // lines)	
(2) Find:	∠ACD = 70˚	

The reader will notice that the presentation steps shown so far are deliberately crafted to make explicit the processes involved in proof strategies: (1) State "Given", (2) state "Conclusion", (3) Use diagrams to map out overview of 'attack', (4) Translate to text, and (5) Provide warrants. Thus, if carried out successfully, the solution presentation of Problem 1a serves as a good preparation for proof in the similar Problem 1b.

Applying the steps to "Problem to prove"

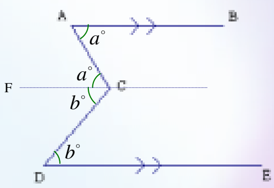
After the solution presentation of Problem 1a, Problem 1b can be shown to students as a form of extension of the earlier problem. The 'template' constructed earlier for Problem 1a can also be used (as shown below). Notice the key difference between the templates: the change from "(2) Find" to "(2) Prove". Apart from mentioning the difference of terms, this is also a good opportunity for teachers to highlight to students that while the process of presentation is similar, the fundamental difference between 1a and 1b is that in proof problems, the conclusion is known, and the main task is in communicating the working very clearly to readers.

(1) Given:	AB // DE D is between AB and DE	(3) Diagrams
(4) Working:		
(2) Prove:	∠ACD = ∠BAC + ∠EDC	

The same steps in the solution process of Problem 1a can essentially be applied to that of Problem 1b (resulting in the working as shown below). The main challenge for students could perhaps be the absence of numeric data. But given that the procedure is almost identical to the earlier problem, suitable references to the numeric quantities of the previous solution might help surmount the problem of using *a*° in place of 30° and *b*° in place of 40°.

[1] For a longer discussion on the difference between "problems to solve" and "problems to proof", I refer the reader to my earlier article (Leong, 2007) in Math Buzz.

(1) Given:	AB // DE D is between AB and DE	(3) Diagrams
(4) Working:	∠ACF = a ° (Alt. angles, // lines) ∠DCF = b ° (Alt. angles, // lines) Therefore, ∠ACD = a ° + b ° ∠ACD = ∠BAC + ∠EDC	
(2) **Prove:**	∠ACD = ∠BAC + ∠EDC	



Summary

Most students will find it difficult to learn geometry proofs solely within the 2-3 weeks allocated in the Upper Secondary Additional Mathematics Syllabus. My recommendation is to slip in the prove-mentality in other occasions where geometry is taught, including at Lower Secondary levels. As I attempt to demonstrate above, problems to prove need not be very challenging; its working need not be long and it does not necessarily involve complicated mathematics. We can start with simple, familiar problems (such as Problem 1a) and inculcate in students a habit to extend to proofs of more general results. Admittedly, a one-off presentation of Problem 1a and Problem 1b will not be adequate. These problems are intended to serve as illustrations of the process of developing prove-mentality via suitable stages. I encourage teachers to find other occasions in your geometry teaching to slip in problems similar to 1a and 1b that can help students develop the prove-mentality over time. I end this article with some possible Problems 2b, 3b, and 4b. I leave the readers with the challenge to develop the corresponding Problems 2a, 3a, and 4a.

Problem 2b: Prove that opposite angles of a parallelogram are equal

Problem 3b: Prove that the sum of the interior angles of an n-sided convex polygon is (n - 2) x 1800 .

Problem 4b: Prove that the area of a kite is half the product of its diagonals.

Reference:

Leong, Y. H. (2007). Difficulties with geometry proofs. Maths Buzz 8(1), 4-6. Singapore: Association of Mathematics Educators.

The ideas in this article were first presented in a workshop conducted during the Mathematics Teachers Conference 2008. I thank the teachers who attended the workshop. The positive feedback from some of the participants gave me the encouragement to share the ideas to a wider audience through this magazine.

2nd Inter - School Sudoku Challenge

Va sughi d/o Kothandapou
Broadrick Secondary School

Many secondary schools students show keen interest in solving the Sudoku Puzzles published in Today's Paper. Although the Sudoku Puzzles are not directly related to the Mathematics taught in our classroom, these puzzles helps to stretch students' thinking skills and encourages them to persevere.

On 23rd July 2008, Broadrick Secondary School organised the 2nd Inter-School Sudoku Challenge. Total of 23 schools from various zones took part in this competition. The Guest of Honour was Ms Theodora Tan, E5 Cluster Superintendent. Even before the start of the competition, many of the students were practicing seriously using the Sudoku Books that were issued as door gifts.

The current Mathematics Framework for Secondary School comprises 5 main domains, namely

1. Attitudes
2. Metacognition
3. Processes
4. Concepts
5. Skills

Mathematical problem solving encompasses all the 5 domains. Concepts, Skills and Processes are normally taught in the classroom. However, Attitudes and Metacognition needs to be cultivated in and beyond the classroom context. Hence, the Sudoku Challenge is an avenue to help students build up these two domains.

Although the rules of the game are simple, the puzzles test students' logical reasoning skills. The puzzles are usually ranked according to their level of difficulty, the most difficult one requiring advanced reasoning skills and strong problem solving skills. These skills are important in Mathematics.

The top 4 winners for this competition were
Queensway Sec School (1st)
Teck Whye Sec School (2nd)
Yu Ying Sec School (3rd)
Tanglin Sec School (4th)

The following schools won the consolation prizes
Broadrick Sec School
Chestnut Drive Sec School
Clementi Town Sec School
Marsiling Sec School

The prizes and door gifts were sponsored by Modern Montessori, Ask and Learn, and Ace Learning.

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