The Mathematician Educator
2022, Vol 3, No. 2, 101-132

A CTD Lesson Planning Tool for Mathematics Teachers

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Lesson planning is a critical aspect of teachers' planning for instruction. Research has shown that a mathematically robust lesson is one that has mathematics content that is focused and coherent, the cognitive demand of tasks students engage with is appropriate, and students have agency, authority and identity as learners. This study proposes a planning tool, for a lesson or sequences of lessons, that focuses on the core of mathematically robust lessons, i.e., the mathematics (content), tasks and discourse. The tool was found to facilitate the development of teachers, both novices and experts.

Keywords: lesson planning tool, mathematically robust lessons, expert teachers, novice teachers, development of teachers

Introduction

Lesson planning is an important process that teachers partake prior to enacting a lesson so that they have a plan to guide them. Research has shown that the quality of classroom instruction is influenced by teacher planning (Clark & Yinger, 1987). A lesson plan functions as an aid for teachers to see the connection between the curriculum and practice (Clark, 1988). During lesson planning, teachers amalgamate syllabus guidelines, ministry initiatives and their own beliefs into a course of action for their lessons (Calderhead, 1984). In a typical lesson plan, teachers outline and describe what students will learn during the various sections of the lesson and how their progress will be measured. In the last revision of the secondary school mathematics syllabus carried out by the Ministry of Education (MOE) in Singapore in 2020, emphasis was placed on teaching towards Big ideas in mathematics (MOE, 2019). It has been noted that teaching towards Big ideas is pedagogically challenging for teachers (Mitchell et. al., 2017).

Beginning teachers are first introduced to the writing of lesson plans as part of their pre-service teacher education. Often these teachers adopt generic templates to write their plans that focus on a few main concerns (Lee, 2009):

1. How would I introduce the lesson?
2. What mathematical concepts or skills would I develop in the students?
3. How would I check for student understanding?
4. How would I conclude the lesson?

These templates often lack a push for deeper thinking and framing of mathematical tasks that teachers may engage students with so as to enact mathematically robust lessons. In the context of this paper, a mathematically robust lesson is one that places emphasis on mathematical ideas (the content), tasks (activities that students would engage with) and discourse (both teacher and student talk about mathematics). Moreover, lesson planning is often deemed to be more relevant and useful for pre-service and beginning teachers, and less so for experienced teachers. There is a need to dispel this myth as lesson planning tools support ongoing teacher development.

In Singapore, there is a need for mathematics teachers to incorporate more dialogic math talks in their classroom discourse (Kaur et al., 2019). Dialogic math talks involve discussion and dialogue (Alexander, 2018). Also, more efforts need to be made to include rich and interesting mathematical tasks that provide students with opportunities to learn mathematics as a connected body of knowledge and to increase student motivation (Kaur et al., 2019). To support teachers in planning mathematically robust lessons, a lesson planning tool may be needed. The study reported in this paper has two objectives.

The first objective is to draw on existing literature and propose a lesson planning tool that mathematics teachers may adopt as a guide to plan mathematically robust lessons. This tool will be situated within the Singapore School Mathematics Curriculum Framework, and will guide the teachers through prompts on examining i) Content: What am I teaching?, ii) Task: What can I use?, and iii) Discourse: How can I facilitate learning? The second objective is to trial the tool and document its efficacy, making possible revisions if necessary.

Review of Literature

Mathematically Robust Lessons

A mathematically robust lesson is a lesson that focuses on the domains of learning identified by Anderson and Krathwohl (2001), namely the cognitive, psychomotor, and affective domains. The cognitive domain of learning includes the development of knowledge and skills (that are algorithmic in nature), the psychomotor domain of learning includes the development of skills involving the use of mathematical tools, such as the protractor, compasses etc., while the affective domain of learning includes attitudes, feelings, and motivations.

Content: Concepts, Skills and Big Ideas

The Teaching for Robust Understanding (TRU) framework extensively researched and framed by Schoenfeld (2014) exemplifies five dimensions of mathematically powerful classrooms. These dimensions are shown in Table 1.

Table 1.
The 5 dimensions of mathematically powerful classrooms (Schoenfeld, 2014)

The mathematics	The extent in which the mathematical content and tasks used are mathematically rich.
Cognitive demand	The extent in which students engage in productive struggle.
Access to mathematical content	The extent in which students engage in the content and tasks equitably.

Agency, authority and identity	The extent in which the classroom environment provides opportunities for students to engage in the lesson and be recognised for their contributions, to build positive identities as a mathematics student.
Uses of assessment	The extent in which the teacher uses formative assessment to guide subsequent instruction.

It is apparent from the framework that the learner and the environment are the foci of mathematically powerful classrooms. The first dimension of the framework encapsulates the robustness and coherence of the mathematical content in a lesson. It refers to the concepts and skills that students need to understand and acquire respectively. Concepts and skills are two of the five components of the Singapore School Mathematics Curriculum Framework, shown in Figure 1.

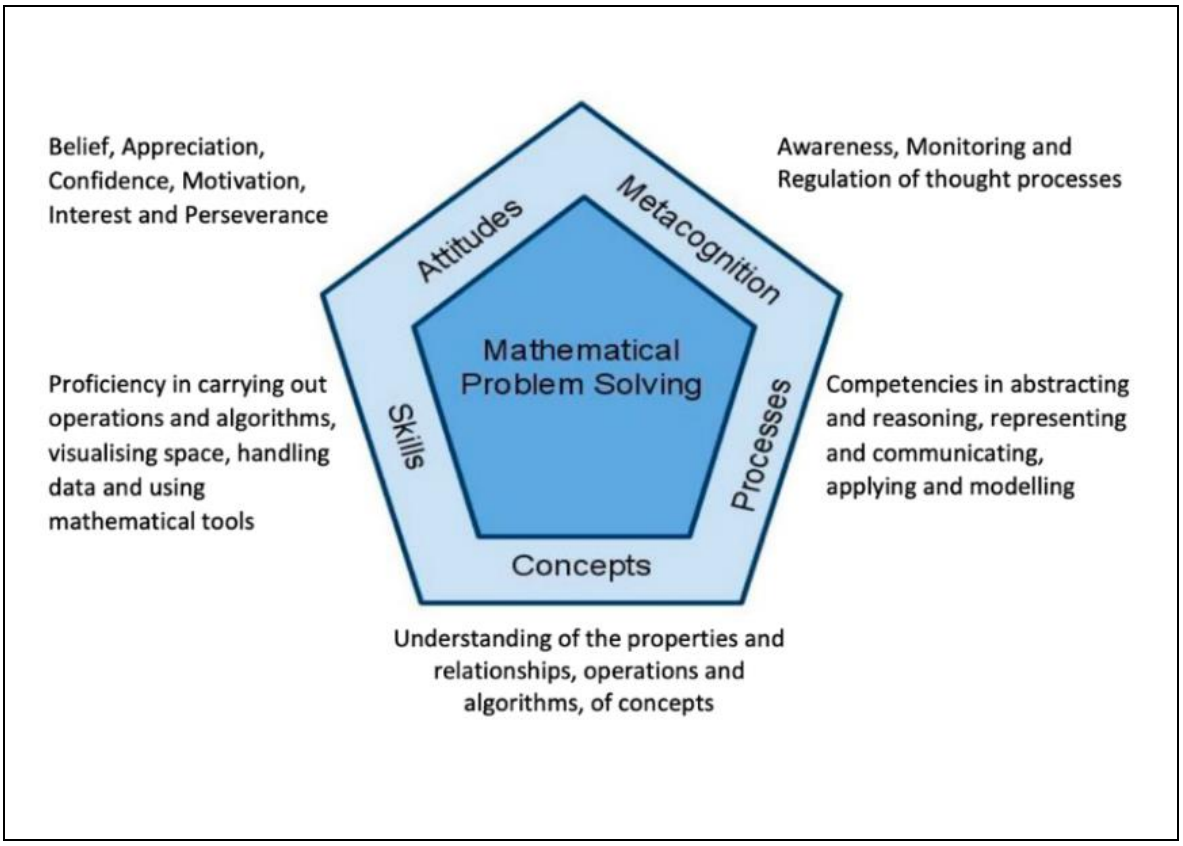

Figure 1. The Singapore School Mathematics Curriculum Framework (MOE, 2019)

The mathematical content is linked by Big ideas. A Big idea is a 'statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole' (Charles, 2005, p. 10). Teaching towards Big ideas was emphasised in the last revision of the syllabus with an aim to bring coherence and demonstrate connections across topics and levels so that students can appreciate the nature of mathematics (MOE, 2019, p. S2-3). Such ideas also lay foundations that facilitate connections between past and new knowledge, thereby deepening students' understanding of mathematics. There are eight Big ideas listed in the syllabus – diagrams, equivalence, functions, invariance, measures, models, notations, and proportionality (MOE, 2019). For teachers to facilitate students' understanding

of concepts, development of skills and linking the content to Big ideas of mathematics, teachers should select appropriate tasks and facilitate rich classroom discourse.

Mathematical Tasks

Mathematics tasks are critical for students to think and reason mathematically. They may consist of a set of problems or one complex problem that directs students to a particular mathematical idea (Stein et al., 1996). Yeo's (2017) classification of mathematical tasks is comprehensive as it comprises tasks for development of concepts (mathematically rich tasks) and practice of skills (non-mathematically rich tasks). The classification is shown in Figure 2.

Mathematically rich tasks provide students with opportunities to learn new mathematical concepts through analytical and synthesis work. Non-mathematically rich tasks provide students with procedural practice on skills that were taught. The selection of mathematics tasks is critical as they impact student learning. However, equally important is *how* the teacher facilitates the classroom discourse to help students understand the content through the mathematical tasks.

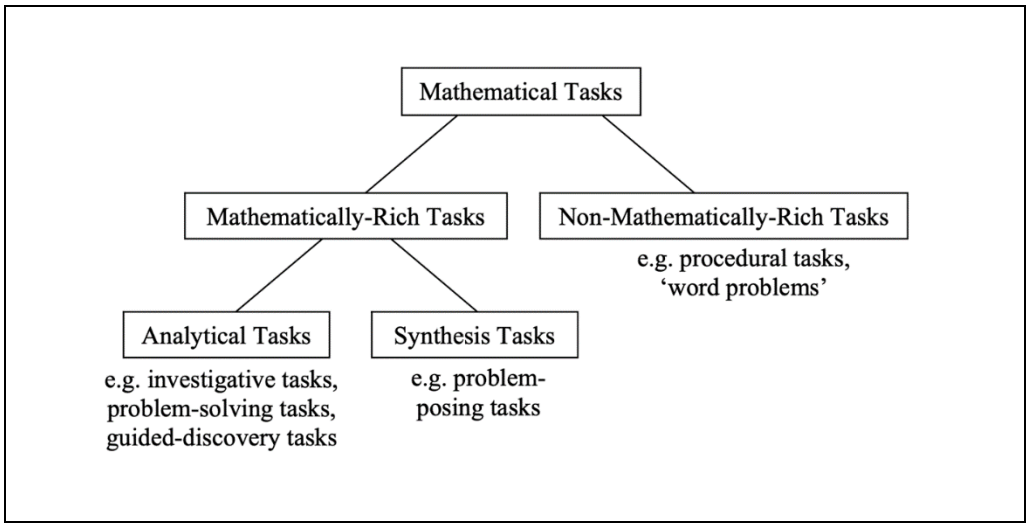

Figure 2. Classification of mathematical tasks according to teaching purpose (Yeo, 2017)

Classroom Discourse and Assessment

Meaningful classroom discourse can enhance student learning. Classroom discourse consists of teaching math talk (from the perspective of the teacher) and learning math talk (from the perspective of the students). Alexander's (2018) dialogic teaching framework places math talk strategies in a continuum between univocal and dialogic. Univocal math talk involves the teacher delivering the exact message and ends when the receiver (student) understands the message. In dialogic math talk, there is an open two-way communication that uses dialogue as a way of thinking to generate meaning in the message.

Teaching math talk strategies consist of rote, recitation, instruction, discussion, and dialogue; while learning math talk strategies include narrate, explain, speculate, imagine, explore, analyse, evaluate, discuss, argue, justify and question (Alexander, 2018). It is important to note that all teaching math talk strategies are equally important, and each serves different

instructional goals in different classroom contexts. Dialogic teaching involves all five teaching math talk strategies, but with more emphasis on discussion and dialogue. In Singapore classrooms however, there is a distinct lack of dialogic math talk (Kaur et al., 2019).

Smith and Stein (2011) introduced a framework consisting of five practices to orchestrate mathematically productive dialogue that focuses on student thinking. The five practices are anticipating students' responses before the lesson, monitoring students' responses during the lesson, selecting students to share, sequencing the student presenting, and connecting solutions and key mathematical ideas. All of these practices aid the teacher in planning, orchestrating and executing productive mathematical dialogue.

Of particular importance is that of anticipating. Anticipating requires the teacher to plan before the lesson what students will do when faced with a task, how to respond to their approaches, and identify strategies that aim to maximise student learning (Smith & Stein, 2011). This is similar to Planning Key Questions in the Singapore Teaching Practice and is critical as it prepares a teacher to be able to respond to the various approaches and questions that the class might have, and skillfully manoeuvre the classroom discourse accordingly.

There are several moves that teachers can use to lead classroom discourse. They may choose to revoice students' responses, ask students to restate or compare reasonings, invite further participation or use wait time (Smith & Stein, 2011). All these facilitate students to engage in mathematical dialogue with the teacher and with one another.

However, for classroom discourse to be effective, students must participate actively. Schoenfeld's (2014) TRU framework suggests that students must be given opportunities to contribute to classroom discourse so that they develop the capacity and willingness to participate (agency) and gain recognition for being good in mathematics (authority), resulting in them developing positive identities as students of mathematics. In addition to these, students should develop intrinsic motivation and the joy of learning mathematics. Intrinsic motivation plays an important role in student achievement and learning (Lim, 2010). In Singapore, there is a nation-wide push towards inculcating the joy of learning amongst students. The joy of learning is an intrinsic motivation in students that drives them to explore their interests, develop their passion and love what they do (MOE, 2017b). Kok (2018) suggests two ways to foster the joy of learning mathematics. The first is to place students at the center of learning, much like what the five dimensions of Schoenfeld's TRU framework emphasises. The second is to provide students with thought-provoking challenges to drive them to further exploration. Wang et al. (2017) proposes that one way the joy of learning can be nurtured is to provide students with the rationale of the task. When planning for lessons, teachers should frame the learning in real-life contexts that students can identify with to pique their curiosity and interest. This increases the intrinsic motivation within the students to learn and engage in the task. To cultivate the joy of learning, teachers should be cognizant of learning experiences that simultaneously interest students in the relevancy of mathematics to their lives and challenge students within their zone of proximal development (Vygotsky, 1978).

Classroom discourse is also a form of informal formative assessment. Informal formative assessment is based on the premise that everyday classroom activities provide opportunities for teachers to collect evidence and assess student learning (Ruiz-Primo, 2011). By promoting meaningful classroom discourse through dialogues and discussions, teachers gain insights on

students' learning and understanding, and in turn these enable teachers to plan for subsequent scaffolding or instruction.

It is important to ensure that the relevant mathematics vocabulary is introduced and explained thoroughly to ensure that classroom discourse is precise and productive. Mathematics vocabulary plays a key role in helping students link symbolic and visual information when performing mathematics tasks (O'Halloran, 2005), and in turn, affects mathematics performance. Peng and Lin (2018) demonstrated that mathematics vocabulary consists of subtypes, such as measurement, geometry and numerical operation vocabulary, and they each relate differently to mathematics performance. Their research suggests that explicit instruction of mathematics vocabulary may be important, especially in instruction involving problem tasks. Hence, when planning for the lesson, teachers should identify important keywords and phrases that students must understand prior to performing the mathematical tasks and ensure that adequate instruction is given to students. This includes keywords and phrases that students are going to learn (new content) and have learnt (prior knowledge).

The Singapore Teaching Practice

In 2017, MOE launched The Singapore Teaching Practice (STP) (MOE, 2017a), a model that illuminates how effective teaching and learning is attained in Singapore schools. In this model, a set of pedagogical practices customised for Singapore classrooms is described. As shown in Figure 3, the pedagogical practices are classified into four teaching areas – positive classroom culture, lesson preparation, lesson enactment, and assessment and feedback.

Lesson Preparation

Of direct relevance to lesson planning are the seven pedagogical practices that are part of lesson preparation. The seven pedagogical practices are described in the following.

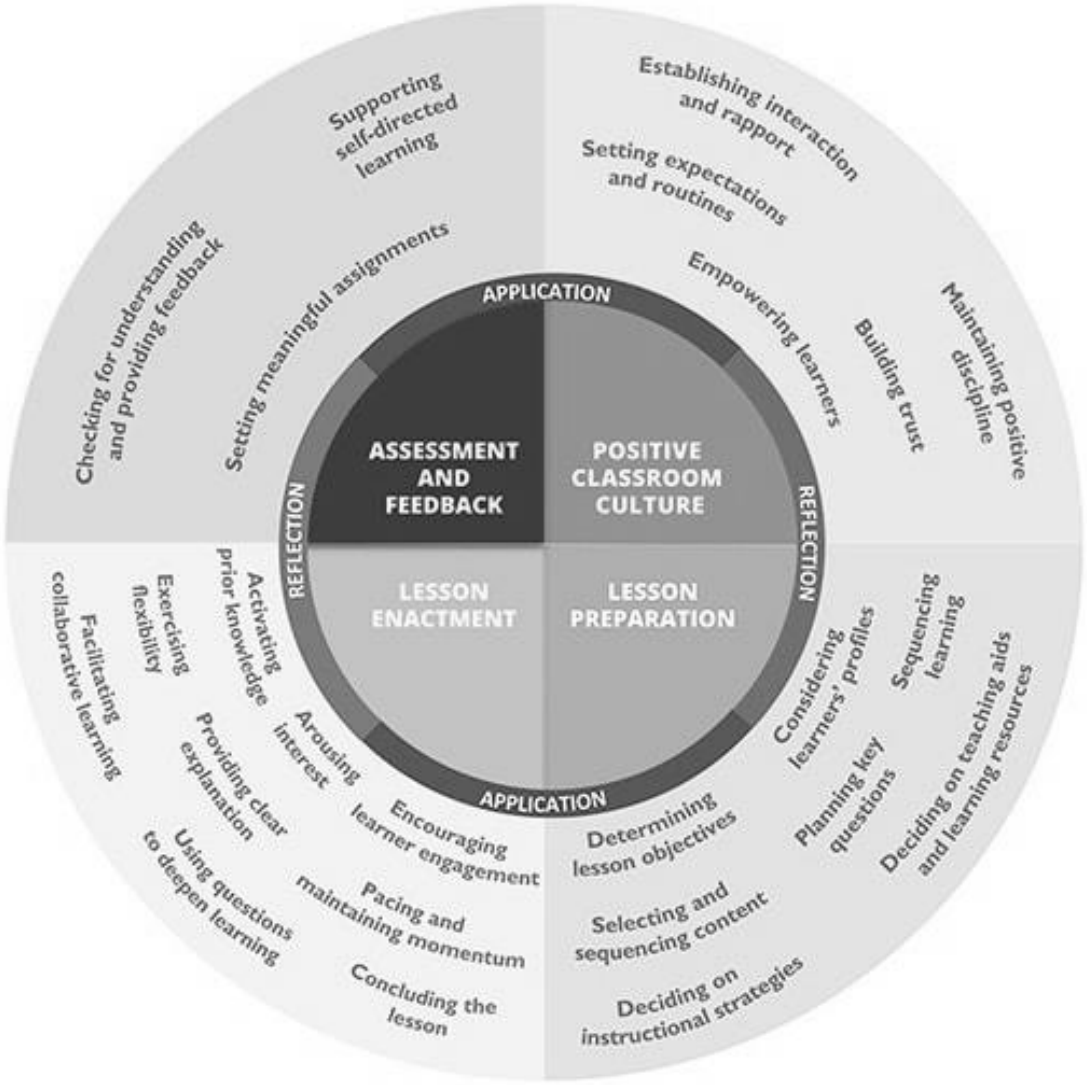

Figure 3. The Pedagogical Practices (MOE, 2017a)

- Determining lesson objectives

The first key step of lesson planning is to identify the learning objectives. Learning objectives are what teachers want students to accomplish by the end of the lesson. They function as a roadmap for teachers to guide instruction and take into consideration the concepts and skills that students have to co-construct and acquire during the lesson. Wiggins and McTighe (2005) also advocate identifying learning objectives as the first step of lesson planning in their three-stage backward design lesson planning approach. With these learning objectives in mind, teachers may then determine acceptable manifestations of student learning and plan instruction accordingly. To identify the learning objectives, the teachers would have to draw on their knowledge of content and curriculum (KCC) (Ball et al., 2008).

- Selecting and sequencing content

Appropriate sequencing and scaffolding of content help students learn in a coherent and cohesive manner. Selecting and sequencing content include considering the content to be taught, the syllabus and the profile of students. It includes breaking down content into

meaningful parts so that students can understand them (Orlich et al., 2010). It also includes differentiating the instruction according to different student profiles.

Differentiated instruction (DI) involves teachers modifying instructional experiences to cater to unique student characteristics, with the aim of increasing students' access to the content (Tomlinson, 1999). However, DI should not compromise on the cognitive demand and rigour of mathematical tasks, and students must be given opportunities to engage in productive struggle. Students engage in productive struggle when they grapple and think critically about concepts, procedures and ideas (Hiebert & Grouws, 2007). To provide such opportunities, teachers may ask meaningful questions and stimulate critical thinking in a safe space for students to struggle (Warshauer, 2015).

Lynch et al. (2018) proposed a set of questions that teachers should consider to maintain productive struggle in an inclusive classroom. The questions are:

- What are the underlying mathematical principles in the task?
- How can each student gain access to the context of the task?
- How can each student engage in the task in a way that builds on his/her prior knowledge?
- How can the teacher provide feedback in the form of questions?
- What discussions should take place to help support students' understanding?
- How much time should the teacher give students to engage in the task?

o Considering learners' profiles

To focus instruction on the students, teachers should first consider the profile of the students. Amongst the things to consider are the students' strengths, weaknesses, preferences, needs, interests, and readiness to learn (Powell & Kusama-Powell, 2011). Information on the profile of students might not be readily available, especially for teachers who are new to the students. An effective way to learn more about the students is to build strong teacher-student relationships (TSR). Teachers may establish rapport with the students by showing care and respect and being friendly and approachable. Positive TSR can amplify learning by increasing students' effort and engagement (MOE, 2017a), and inform teachers on the profile of the students to categorise learning for them.

o Deciding on instructional strategies

There are a multitude of instructional strategies available to teachers that serve different learning goals and needs. An appropriate instructional strategy can enhance engagement of students, develop deep understanding of the content, and help students learn effectively. Instructional strategies include demonstration, direct instruction, inquiry, small group discussions, experiments, problem-solving, guided practice, and projects.

o Deciding on teaching aids and learning resources

Mathematics tasks can be obtained from a variety of resources. When planning for a lesson, teachers should select appropriately from a wide variety of learning resources to support students' learning and consider how well those resources serve the intended teaching purpose and learning objectives. This requires teachers to draw on their knowledge of content and teaching (KCT) (Ball et al., 2008). KCT can be developed through experience, collaboration with peers and/or attending professional development courses aimed at improving teachers' familiarity and competency with a variety of learning resources.

- Sequencing learning

Sequencing learning involves ordering tasks within and across lessons. It affects how students process, consolidate, and apply concepts and skills. When sequencing learning, teachers should always consider the lesson from students' perspectives. They should consider students' abilities and interests to enact lessons that engage students. Also, sequencing learning entails scaffolding tasks meaningfully, such as from simple to complex concepts, or from concrete to abstract etc. Sequencing and scaffolding tasks require teachers to draw on their KCT and knowledge of content and students (KCS) (Ball et al., 2008) respectively.

- Planning key questions

Questions are essential components of dialogic math talks (Alexander, 2018). Effective questioning promotes student involvement and engagement, and guides students cognitively and metacognitively to achieve deep and meaningful learning. Hence, it is important that teachers plan key questions to ask during the lesson, with each question serving as a key checkpoint of students' learning and understanding. When planning key questions, teachers should consider the purpose of the question and the type of question to ask. The purpose frames the type of question to be posed, and is related to the cognitive processes the teachers hope to engage the students in. These processes include remember, understand, apply, analyse, evaluate, and create (Anderson & Krathwohl, 2001). Teachers should also anticipate the various responses students may offer. When responding to the responses, teachers may choose to praise, encourage, prompt, clarify or address the students' understanding (MOE, 2017a).

Lesson Enactment, Assessment and Feedback

In addition to the seven pedagogical practices listed under lesson preparation, some of the other pedagogical practices listed under lesson enactment, assessment and feedback too require planning by the teacher (See Figure 3). To support lesson enactment and assessment, teachers need to also consider the following three pedagogical practices as they support student learning and are important considerations when planning a lesson.

- Activating prior knowledge

Students' prior knowledge consists of the content, concepts, and skills that they have previously learnt. Students have varying levels of prior knowledge of the mathematical content before the start of a lesson. During a lesson, they make meaning of new content by assimilating the new knowledge with their prior knowledge. Hence, it is important that teachers activate prior knowledge before teaching new content on a topic by guiding them through a reflection of what they already knew about the topic. This may also address any learning gaps that they might have in previous stages of study.

- Arousing interest

When students are interested in the content, they will be engaged in the lesson and intrinsically motivated to learn. Hence, it is important to arouse students' interest in the content.

- Checking for understanding

Checking for understanding is an important pedagogical practice consisting summative and formative assessments. Summative assessments include homework, assignments, quizzes, tests, and examinations. There are many ways to assess students' understanding formatively. Classroom discourse is an informal formative assessment that provides insights on how much students understood from the lesson. It enables the teacher to plan for subsequent instruction

and improve teaching practices. Through engaging in dialogic math talk, the teacher may check for students' understanding of the content by identifying any learning gaps and close any learning gaps by providing appropriate feedback.

Mathematics Specific Pedagogy of Teachers

It was Shulman (1987) who coined the term "pedagogical content knowledge" in an attempt to capture the knowledge that teachers need to facilitate teaching and learning. Ball (2002) expanded this idea of content specific knowledge for teaching of mathematics, labelling it "mathematical knowledge for teaching" (MKT). Mathematical knowledge proposed by Hill et al. (2005) comprises six strands as illustrated in Figure 4.

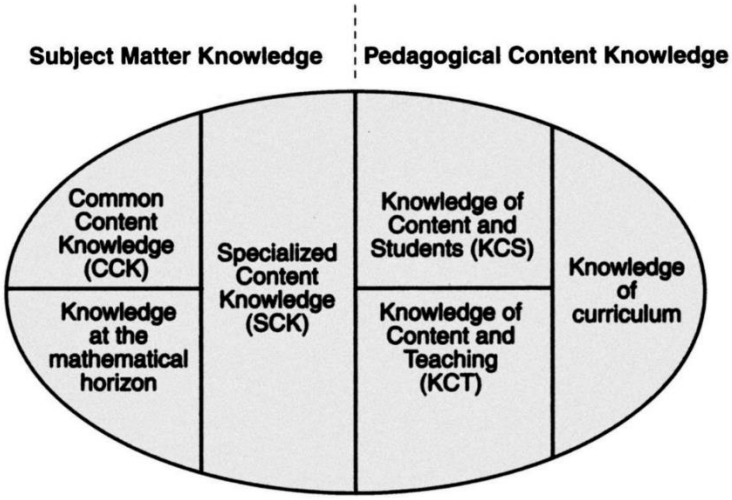

Figure 4. Domains of mathematics knowledge for teachers

Teachers' KCS enables them to understand how students learn particular content (Ball et al., 2008). This knowledge influences teachers' thought processes in considering which strategies and tasks would resonate with students and how students will respond to them. Teachers' specialised content knowledge (SCK) allows them to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures as well as to examine and understand unusual methods to problems (Ball et al., 2008). This knowledge propels teachers to make important decisions when adapting tasks to cater for the learning specificities of students to effectively address students' misconceptions and gaps in conceptual understanding. Also, of significance is KCT that guides teachers in sequencing content for instruction, and how to evaluate instructional advantages and disadvantages of particular representations (Hill et al., 2005). Teachers' possession of this knowledge inexorably and directly impacts their capabilities in manipulating tasks considering the scaffolding and structure the students require. A lesson often comprises several phases, like introducing a concept, developing conceptual understanding, practice leading to procedural fluency and even the addressing of misconceptions and errors. For each of these phases, teachers select suitable tasks matching their specific lesson goals. According to the nuances of the phases, teachers draw on their MKT when selecting tasks for these varied purposes during planning.

Lim-Teo et al., (2011) noted that discipline-specific pedagogy affects classroom practices, which in turn affect learning by students. They advocated checklists comprising observable mathematics teacher practices categorised as (i) choice and sequencing of activities, (ii) connections between topics and between concepts, (iii) balance between concept development and mathematical procedures, (iv) classroom talk – explanation, questioning, responding to students, as tools for mathematics teacher leaders to develop mathematics teachers in their schools.

Novice and Expert Teachers Lesson Planning

Housner and Griffey (1985) noted that when planning lessons, novice and expert teachers perceived their classrooms differently. Novice teachers tended to regard their class as a whole (an entity – devoid of any individual differences) while experts on the other hand considered their classroom as a collection of unique individuals. As such, experts and novices may place emphasis on differing aspects of a lesson plan.

Borko and Livingston (1989) also found distinctions among expert and novice mathematics teachers in scripting their lesson plans. Novices tended to plan lessons as bite-sized scripts while expert teachers preferred to plan for sequences of lessons of a unit of instruction. In addition, experts often kept written planning to a minimum, highlighting the main components of the lesson while storing the remaining part of the lesson mentally. Learning how to plan can be a feature of every teachers' learning well beyond their pre-service teacher education, as it is through planning that teachers are able to learn about teaching and through teaching that they are able to learn about planning (Mutton et al., 2011).

The Tool and its Efficacy

Research Design

A pragmatist paradigm is adopted for the study as the reality of teachers' work related to curriculum is constantly re-negotiated, debated and interpreted (Kuhn, 1970). In addition, design-based research informs the design of the study as (i) the study is situated in a real educational setting, and (ii) the design of the tool is guided by practice of a practitioner (the researcher), is informed by relevant literature, and (iii) the tool (intervention) is trialed for its efficacy (Anderson & Shattuck, 2012).

A Review of Lee's (2009) Lesson Planning Template

During pre-service teacher education in Singapore, mathematics student teachers are introduced to Lee's (2009) lesson planning template. The components in the template are presented in Table 2.

Table 2
Lesson plan template in Lee (2009)

	Components	**Elaboration**
1.	Content	The mathematics concept, rule, skill, law, principle or knowledge that the teacher intends to teach.
2.	Specific instructional objective	What students must be able to demonstrate at the end of the lesson.
3.	Prerequisites	What the student must know or be able to do before the lesson.
4.	Instructional procedures	Description of the introduction, development of concepts, application of concepts and closure of the lesson.
5.	Materials and equipment	List of all the resources required for the lesson.
6.	Extra material	Additional resources that might be needed in the event of leftover time in the lesson.
7.	Self-assessment	Teacher's reflection of the lesson
8.	Homework	Carefully chosen homework questions for students to complete

For teachers to consider Big ideas when planning their lessons, Lee's (2009) lesson template was first reviewed. The review was guided by three aspects of mathematically robust lessons. The aspects are i) *Content:* What am I teaching?, ii) *Task*: What can I use?, and iii) *Discourse*: How can I facilitate learning?

The Content

In Lee's lesson template (see Table 2), the components – content, specific instructional objective, prerequisites, and instructional procedures include aspects of the mathematical content that would be worked on during the lesson. As part of the instructional procedures, teachers are advised to detail how they would Introduce, Develop, Consolidate and Close the lesson. For lesson closure, teachers are pushed to think about how to "prepare students for the next lesson, such as a follow-up activity to reinforce and extend the lesson" (Lee, 2009, p. 342). Though the template does help the teachers to consider the content appropriately, it lacks explicit guidance for teachers, in general, to consider Big Ideas in school mathematics, as emphasised in the school mathematics curriculum for Singapore schools (MOE, 2019). The consideration of Big Ideas would engage teachers in facilitating students developing their mathematics as body of connected knowledge. Therefore, there is a need for teachers to consider past, present, and future knowledge of mathematics for students.

Future knowledge refers to ideas for which the content is a subset and prerequisite of, and includes Big Ideas, related advanced concepts, and the application of the concepts to real life situations and other subject disciplines. Big Ideas anchor students' learning to develop deeper understanding of mathematics (Charles, 2005). Related advanced concepts serve as a preview of the exciting mathematics yet to be learnt. Application of the concepts to real life situations and other subject disciplines can arouse students' interest when they see the relevancy of what they are learning.

The prior knowledge, present content and future knowledge give a complete view of how the mathematical content to be taught situates within the span of the curriculum. It allows teachers to see how current content builds on what was learnt previously by the students, and how it serves as a building block for future learning. Teachers can thus have a good grasp of the content to be taught in the lesson and understand mathematics as an inter-connected body of

knowledge. Therefore, it is imperative that teachers should plan carefully how to enable students to see and appreciate the connections between the three types of knowledge: prior, present, and future.

The Task

In Lee's (2009) lesson plan template, the fourth to sixth components on instructional procedures, materials/equipment and extra materials relate to the tasks of the lesson. As these components are again rather general, teachers may not focus on the distinctions among, (i) learning and practice tasks (Kaur, 2010), (ii) learners' profile (Powell & Kusama-Powell, 2011), and (iii) DI (Tomlinson, 1999) during the lesson planning process.

Mathematical tasks form the bedrock of mathematics instruction. Mathematically rich tasks aid in the *learning* of concepts, while non-mathematically rich tasks aid in the *practice* of skills (Yeo, 2017). When planning the instructional procedures, the development of concepts involves learning tasks, while the application of concepts involves practice tasks (Kaur, 2010). Understanding the distinction between these two types of tasks helps teachers to identify and select tasks suited for the lesson, depending on the learning objectives.

Considering the profile of students and catering instruction accordingly is an important step in lesson planning (MOE, 2017a; Orlich et al., 2010). In the event of any mismatch between instruction and student profiles, students might not be interested in participating in the lesson, impeding on their learning. Amongst the things to consider are the students' strengths, weaknesses, preferences, needs, interests, and readiness to learn. These knowledge about students will help teachers in deciding how to select and sequence instruction to maximise the effectiveness of the lessons.

Different students learn at different paces. Hence, it is important that teachers think of how to differentiate instruction to allow every student to learn at an appropriate and suitable pace (Tomlinson, 1999). However, it is important that DI maintains the cognitive demand and rigour of the mathematical tasks, enabling students to engage in productive struggle at the appropriate level.

The Discourse and Assessment

In Lee's (2009) lesson plan template, the eighth component on homework is on assessment. While there are no components on discourse, many teachers typically include key questions and anticipated students' responses when planning the instructional procedures (Component 4). During lesson planning, teachers may not focus on the explicit planning of classroom discourse, how to motivate students to participate in classroom discourse, and how classroom discourse can aid as a formative assessment tool (Ruiz-Primo, 2011).

Classroom discourse is an important component of a mathematically robust lesson as it can enhance student learning when done meaningfully (Schoenfeld, 2014). Planning for classroom discourse includes not only key questions and anticipated students' responses, but also the introduction and usage of appropriate mathematical vocabulary (O'Halloran, 2005). As noted by Alexander (2018), as part of classroom discourse, all math talk strategies are equally important as they serve different instructional goals in different classroom contexts. Hence, it is important for a teacher to decide on the math talk strategies to use and how to use them appropriately in the lesson.

Classroom discourse requires active participation between the teacher and the students. Hence, planning for rich classroom discourse requires intentional planning to engage the students and motivate them to participate (Kok, 2018; Wang et al., 2017). Teachers should intentionally nurture a safe environment for discourse and provide equal opportunities for every student to develop his or her capacity and willingness to contribute (Schoenfeld, 2014). Teachers should also strive for students to be intrinsically motivated to participate actively by finding tasks that are interesting and relevant.

It is common for teachers to allocate homework questions targeting repeated application of concepts to assess students' understanding of the concepts. However, classroom discourse can function as an important formative assessment of students' understanding. Through classroom discourse, teachers can gain valuable insights on students' current level of understanding, and plan for subsequent instruction (Ruiz-Primo, 2011). Hence, teachers should think about the various verbal indicators from students that demonstrate their understanding, or lack of understanding, to assess students' understanding 'on-the-fly' and make appropriate subsequent instructional decisions (Schoenfeld, 2014).

CTD Lesson Planning Tool

The tool, shown in Table 3, builds on Lee's (2009) lesson planning template. It comprises prompts that guide teachers in the planning of a mathematically robust lesson by engaging them to think deeply about all the three aspects: content, task, and discourse. The prompts are guided by literature review and are divided into two categories – fundamental considerations of the content, task and discourse aspects, and deeper considerations involving the interactions of the three aspects. Fundamental considerations help the teachers to focus on each of the three aspects, while deeper considerations help the teachers to focus on the interactions amongst the three aspects.

Table 3.
CTD Lesson Planning Tool

Fundamental Considerations (Level 1)	**Core Aspect(s)**
What am I teaching? 1. What are the main concepts that students co-construct and skills that students acquire during the lesson? 2. What prior knowledge is needed for students to understand the lesson? 3. How is the content related to any Big Ideas, future content in later stages of study and other disciplines?	Content
What can I use for teaching and learning? 4. What learning tasks can I use to get students to understand the mathematical concepts? 5. What practice tasks can I use to ensure students develop the mathematical skills? 6. What real-life applications of the content can I bring into the lesson?	Task
How can I facilitate learning? 7. What are some key questions/prompts I should pose? 8. What are some important keywords/phrases that students must understand?	Discourse
Deeper Considerations (Level 2)	**Core Aspect(s)**
How can I focus students on the mathematics? 9. How should I sequence and scaffold the learning tasks and practice tasks? 10. How can I demonstrate/make apparent/elicit the connections between the content, the prior knowledge and the horizon knowledge?	Content and Task

How can I maintain the cognitive demand and ensure productive struggle in students? 11. What are some considerations I have regarding the profile of the students? 12. What are some anticipated students' responses and how do I respond? 13. How can I differentiate the learning?	Content and Discourse
How can I ensure that all students have agency, authority and identity in the construction of knowledge? 14. What instructional strategies can I use to facilitate classroom discourse? 15. What kind of activities do the students prefer? 16. How can I motivate the students?	Task and Discourse
How do I use classroom discourse as a formative assessment of students' understanding? 17. What are the verbal indicators to show that students have constructed the main concepts and acquired the skills?	Content, Task and Discourse

The prompts of the tool are not sequential, i.e., teachers need not address each at a time from 1 to 17, but rather they should form the basis for deliberations in crafting a mathematically robust lesson. Such deliberations may be amongst pairs or groups of teachers working together in planning lessons. However, Prompts 1 to 3 on content sets the context for the lesson, so they should be the starting prompts for the lesson planning process. Some of the prompts can and should be considered in tandem, and teachers are encouraged to do so. For example, Prompt 7 on key questions and Prompt 12 on anticipated student responses are related, and it should help the teachers to consider both at the same time for the lesson planning process to be coherent. A simple template was designed for teachers to fill in their responses to the prompts should they require one.

The Prompts

- Content: What am I teaching?

Prompts 1, 2 and 3 direct teachers to consider deeply the mathematical content that they are going to be teaching, and to situate the mathematical content within the curriculum (Charles, 2005). By identifying the learning objectives and how it fits within the span of mathematics curriculum, i.e., how it builds on what was learnt previously and how it is a building block for future learning, teachers will have a good grasp of the content to be taught in the lesson and understand mathematics as an inter-connected body of knowledge. Although it has been suggested that one should not approach the tool in a linear manner, nevertheless, it is necessary for Prompts 1 to 3 to be considered first as they provide the foundation on which decisions on task and discourse can be made.

Prompt 1 guides teachers to identify clearly what are the main concepts they would like to co-construct with the students and what skills they hope students will acquire during the lesson. This will provide the 'end goal' of the lesson and guide their planning accordingly in the subsequent prompts. An important note here is the use of the word 'co-construct'. Rather than presenting the concepts through teacher-directed instruction, teachers are encouraged to plan for lessons with the co-construction of knowledge in mind, building and developing knowledge collaboratively with students in the lesson.

Prompt 2 ensures that teachers are mindful of the varying levels of prior knowledge at which students enter the lesson. This would help teachers identify the gaps and address them before teaching the new content. Concepts and skills are linked across levels in Singapore's spiral curriculum, and mathematical content is built on what was learnt in previous years. In order,

to present mathematics as an inter-connected body of knowledge, teachers should always activate prior knowledge at the start of the lesson to establish the links and provide the prior knowledge as an anchor for students to assimilate new knowledge to.

Prompt 3 aims to make teachers cognizant of how the content is situated within the curriculum. The aim of this prompt is to guide teachers to consider how the content is related to any of the Big Ideas, future content in stages of study and other disciplines (MOE, 2019). It helps to present mathematics as an inter-connected body of knowledge and its crucial role in supporting many other disciplines. By being mindful of how the content is related to Big Ideas and future content in later stages of study, the teacher can help students assimilate the mathematical content and help them see the coherence and connectedness of mathematics across different topics and stages of study. Mathematics has an important supporting role in many disciplines, providing the means to quantify ideas and objects. By considering how the mathematical content is linked to other disciplines, teachers can provide important contexts for learning which can be used to engage students.

- Task: What can I use for teaching and learning?

Prompts 4, 5 and 6 direct teachers to consider deeply the tasks to be used in the lesson. After identifying the content, teachers should research and consolidate all possible learning and practice tasks that can be used for instruction. To promote the joy of learning, tasks involving the real-life application of the mathematics content may be included in the lesson as well (Wang et al., 2017). These three prompts can be considered together with other related prompts, such as Prompts 9 and 11.

The aim of Prompts 4 and 5 is to guide teachers to think deeply on what mathematics tasks are suitable for their lesson based on the intended teaching purpose and the learning objectives. These prompts highlight the distinction between learning tasks and practice tasks (Kaur, 2010). *Learning tasks* are mathematically rich tasks that provide students with the opportunity to learn new mathematical concepts. *Practice tasks* are non-mathematically rich tasks that provide students with procedural practice on skills.

Prompt 6 aims to direct the teacher towards exploring how the mathematical content to be taught is relevant to the students' everyday lives, or finding real-life applications of the content that students might be interested in. This drives the teacher to consider instruction from the students' perspective, providing meaning and engagement in their learning. Problems in real-world contexts was introduced in the previous syllabus review to provide students with opportunities to apply the concepts and skills learnt, to appreciate and cultivate an interest in mathematics in them. By bringing such contexts into lessons, students can identify with such contexts and see the value and beauty of mathematics.

- Discourse: How can I facilitate learning?

Prompts 7 and 8 direct the teacher to consider deeply the verbal interactions with students, specifically the questioning techniques and mathematical vocabulary. The aim of Prompt 7 is to guide teachers to think through the questions and prompts that they would pose to engage students in learning. These questions and prompts can serve as important checkpoints in the lessons. When planning key questions, teachers should consider the purpose of the question and the type of question to ask to engage students in specific cognitive processes – to remember, understand, apply, analyse, evaluate or create.

Prompt 8 is designed to help the teachers plan for classroom discourse more explicitly and ensure that students use the correct mathematical language (O'Halloran, 2005; Schoenfeld, 2014). The aim of this prompt is to guide teachers to be aware of the potential difficulties students might face due to linguistic challenges. Explicit instruction of mathematics instruction is important to help students understand important keywords and phrases prior to performing mathematical tasks. The important keywords and phrases include what they have learnt (prior knowledge) and what they are going to learn.

- Content and Task: How can I focus students on the mathematics?

Prompts 9 and 10 direct the teacher to think deeply about the interactions between content and task and should be considered only after task selection (Prompts 4 and 5). These two prompts guide teachers to consider the appropriate sequencing and scaffolding of the tasks for instruction, while ensuring that the tasks fulfill the learning objectives, build on prior knowledge, and prepare students for future learning.

Prompt 9 is adapted from the STP pedagogical practice of sequencing learning (MOE, 2017a). The aim of this prompt is to guide the teachers to think through how to sequence the learning tasks (prompt 4) and practice tasks (prompt 5) to help the students to learn in a coherent and cohesive manner. This prompt should be considered with the profile of students in mind (prompt 11) as it affects decisions made on the sequencing and scaffolding of content.

Prompt 10 is designed to help teachers relay to students their understanding of how the content is situated within the curriculum (Charles, 2005; MOE, 2019). The aim of this prompt is to guide the teachers to think how to demonstrate the connections between the mathematical content, prior knowledge and horizon (future) knowledge. The connections between the mathematical content, prior knowledge and horizon knowledge can be demonstrated, made apparent or elicited by the teachers in a variety of ways. They include 'intra-connections' and 'inter-connections'. Intra-connections are connections within the mathematical content such as multiple representations of a concept or multiple solutions to a problem. Inter-connections are connections beyond the mathematical content such as the Big ideas or brief introduction for what they will learn in future.

There are fine distinctions between the choice of verbs in the prompt. To *demonstrate* is to guide students step-by-step in the learning. To *make apparent* is to guide the students is to help students to understand the mathematical content clearly. To *elicit* is to use a series of questions or activities to draw out the important learning points from the students.

- Content and Discourse: How can I maintain cognitive demand and ensure productive struggle in students?

Prompts 11, 12 and 13 direct teachers to think deeply about the interactions between content and discourse. The prompts guide teachers to ensure that the mathematical content is cognitively challenging yet appropriate for the profile of students through classroom discourse and DI.

Prompt 11 is adapted from the STP pedagogical practice of considering learners' profiles (MOE, 2017a). The aim of this prompt is for teachers to be cognizant of their students' profile as they guide teachers to select appropriate mathematical tasks and sequence the learning effectively (Powell & Kusama-Powell, 2011). Amongst the things to consider are the students'

strengths, weaknesses, preferences, needs, interests and readiness to learn. These can be ascertained by building strong teacher-student relationships. With this knowledge about the students, teachers can approach the other prompts in this tool and plan effectively for instruction that caters specifically for the students. Hence, this is an important prompt that teachers must always consider in the early stages of planning so that the lesson plan is focused on the students.

Prompt 12 is designed to help the teachers plan for classroom discourse more explicitly (Alexander, 2018). After deciding on the mathematical tasks (prompts 4 and 5) and planning key questions (prompt 7), the aim of this prompt is to have the teacher anticipate students' responses and plan how to react to those responses. Anticipating students' responses serves two important functions. Firstly, it enables the teacher to be prepared for as many possible scenarios in class as possible. Secondly, it builds and develops the teacher's SCK and KCS (Ball et al., 2008). By anticipating the students' responses in advance, the teacher can prepare for appropriate and timely interventions if necessary, keeping students cognitively engaged and motivated to learn. There are a few ways the teacher can choose to respond to students – to praise, encourage, prompt, clarify or address the students' understanding. The teacher should keep the feedback formative and engage in dialogic math talk with the students.

Prompt 13 is designed to help teachers think about how to differentiate their instruction (Tomlinson, 1999). The aim of this prompt is to have the teacher think about how to differentiate learning for students of differing characteristics without compromising on the cognitive demand and rigour of the mathematical tasks. All students should be given the opportunity to engage in productive struggle at the appropriate level (Warshauer, 2015).

To maintain productive struggle, the teacher should consider this prompt in tandem with the other prompts (Lynch et al., 2018), such as

- What are the underlying mathematical principles in the task? (Prompt 1)
- How can each student gain access to the context of the task? (Prompts 3 and 14)
- How can each student engage in the task in a way that builds on his/her prior knowledge? (Prompt 2)
- How can the teacher provide feedback in the form of questions? (Prompt 17)
- What discussions should take place to help support students' understanding? (Prompts 7 and 12)
- How much time should the teacher give students to engage in the task?

- Task and Discourse: How can I ensure that all students have agency, authority and identity in the construction of knowledge?

Prompts 14, 15 and 16 direct the teacher to think deeply about the interactions between task and discourse. When performing mathematical tasks, students engage in dialogue with themselves, their peers, or their teachers. Hence, it is important that all students have the opportunity to raise questions, explain his/her ideas, and contribute meaningfully to class discussions (Schoenfeld, 2014). This allows them to build confidence and identity as a student of mathematics.

Prompt 14 is adapted from the STP pedagogical practice of deciding on instructional strategies (MOE, 2017a). There are a multitude of instructional strategies available to teachers that serve different learning goals and needs. The aim of this prompt is for the teacher to decide on an

appropriate instructional strategy from them. Before deciding on an instructional strategy, it is important to base the decision on the profile of students (prompt 11). The chosen instructional strategy should enable the teacher to facilitate learning that accounts for differences in readiness, interest, and preferences amongst the students. Some possible instructional strategies include direct instruction, demonstration, modelling, small group discussions, problem-solving activities, guided practice, reflective inquiry, and projects.

Prompt 15 is adapted from the STP pedagogical practice of arousing interest (MOE, 2017a). The aim of this prompt is for the teacher to choose learning activities that interest the students so that they will be engaged and intrinsically motivated to learn. To arouse their interest, teachers need to cater instruction to the students' profile, identifying their preferences and needs. Another way interest can be aroused is to use real-life applications of the mathematical content in the class to help them see the relevancy of what they are studying.

Prompt 16 is designed to help teachers think about how to motivate their students to participate in classroom discourse (Kok, 2018; Lim, 2010; Wang et al., 2017). The aim of this prompt is to get the teacher to think specifically on how to motivate students so that they are positively engaged in the mathematical tasks and contributing actively to the construction of knowledge. One way to motivate students is to help them achieve success by appropriately sequencing and scaffolding the learning and practice tasks, allowing them to build confidence as they move from one task to the next. Another way to motivate students is to build a positive classroom culture that empowers them to contribute freely to discussions within a trusted community.

- Content, Task and Discourse: How do I use classroom discourse as a formative assessment of students' understanding?

Prompt 17 directs the teacher to think deeply on how discourse can be used as a formative assessment of students' understanding of the content after engaging in the mathematical task. This prompt was designed to help teachers engage in the STP pedagogical practice of checking for understanding using classroom discourse (MOE, 2017a). The aim of this prompt is to get the teachers to consider how to assess students' learning formatively using verbal indicators.

To use classroom discourse as an informal formative assessment, the teacher should identify verbal indicators that show that the students have constructed the main concepts and acquired the relevant skills. The verbal indicators can include students recalling simple facts, applying or generalising concepts to other situations, or asking questions to deepen their understanding.

Efficacy of the Tool

Methodology

This section outlines the methodology undertaken to determine the efficacy of the CTD Lesson Planning Tool and evaluate the second objective of the study guided by research question*: Did the lesson planning tool facilitate the crafting of a mathematically robust lesson? If so, how did it do so? If not, why did it not do so?*

- The participants

Mathematics teachers employed by the MOE and teaching in secondary schools in Singapore were eligible to participate, subject to satisfying the selection criteria. The first criterion is the number of years of teaching experience they have. This was a consideration as experienced

teachers appear to have greater depth of MKT (Hill et al., 2005) than beginning teachers. In addition, novice and expert teachers perceive their classrooms differently (Housner & Griffey, 1985). To obtain data on the efficacy of the CTD Lesson Planning Tool, teacher participants with less than 3 years (novice) and more than 5 years (expert) of teaching experience will enrich the data collected (Patton, 1990).

All teachers in Singapore are trained to teach two curriculum subjects (CS). MOE typically assigns teachers with CS1 related to the teacher's major of study in university. Hence, a teacher with mathematics as CS1 is assumed to have better grasp of the subject than one with mathematics as CS2. Hence, the second criterion for selecting teacher participants in this study was that teachers have mathematics as their CS1. In addition to the first criterion, this further enriches the data collected.

In the recruitment stage, the criteria for teacher participants were made known to School Leaders and interested teacher participants were given a form to indicate their interest for participation. From the pool of interested teachers, two teachers (Teacher A and Teacher B) that best fit the two criteria above were chosen. Teacher A is a male beginning teacher with 1.5 years of mathematics teaching experience. Teacher B is a female with 6.5 years of mathematics teaching experience. Both teachers were trained to teach mathematics as their CS1 subject.

- Method and data collected

The two teachers who satisfied the criteria for participation were asked for their consent to participate. Following which each teacher met with the researcher three times. Table 4 details the activities that took place during the meetings and data collected.

Table 4
Activities and data collected

Meeting	Activity	Data
1	During the first meeting an interview was carried out that sought information on how the teacher plans for his/her lessons. The interview was guided by the following prompts: 1. Do you plan your mathematics lessons? 2. If you do, how do you create your plan? Do you write out the plan? 3. Do you use any template for your lesson plan? If yes, please elaborate. 4. What are the key considerations guiding your plan? Elaborate as completely as possible. Following the interview, teacher was given the following set of instructions: 1. Pick any lesson that you would be teaching in about two weeks' time. 2. Write a lesson plan for your lesson. 3. You may draw on all of your past knowledge and experiences. 4. Please submit a copy of your plan to the researcher at the start of the next meeting.	The interviews were audio-recorded. Pre-intervention interview data was collected.
2	During the second meeting, a copy of the lesson plan written by the teacher was collected. Next the teacher was introduced to the CTD Lesson Planning Tool shown in Table 3. Following which the teacher was given the following set of instructions:	A copy of the pre-intervention lesson plan was collected.

	1. Using the CTD Lesson Planning Tool, review the lesson plan you have written and revise the plan if necessary. 2. You may seek clarifications about the tool from the researcher, if needed. 3. Please submit a copy of your revised plan to the researcher at the start of the next meeting.	
3	During the third meeting, the revised lesson plan written by the teacher was collected. Next the teacher was interviewed. The interview was focused on finding out about the teacher's reception of the CTD Lesson Planning Tool. The interview was guided by the following prompts: 1. Was the lesson planning tool helpful? Please elaborate. 2. What are the areas of your original lesson plan that you revised? Why did you revise those areas? 3. Are there any aspects of the lesson planning tool which you find are lacking? If yes, please elaborate. 4. Would you recommend this lesson planning tool to your colleagues? Please elaborate.	The revised (post-intervention) lesson plan was collected. The interviews were audio-recorded. Post-intervention interview data was collected.

- Data analysis methods

The qualitative data generated by the lesson plans and also the interviews were analysed through content (text) analysis (Erlingsson & Brysiewicz, 2017). In addition, comparative analysis was also adopted for the study of the pre- and post-intervention lesson plans of the teachers (Simiste & Scholz, 2017). This analysis was guided by a set of analytical prompts that were framed by the three areas of concern, Content, Task and Discourse in the CTD Lesson Planning Tool. Table 5 shows examples of how the lesson plans were coded. The codes were as follows: 2 – apparent, 1 – partially apparent, and 0 – not apparent.

Table 5
Examples of Data Coding

Analytical Prompt	0 – not apparent	1 – partially apparent	2 – apparent
Are links to Big ideas considered in the planning?	No mention of Big ideas.	Stating of Big ideas without elaboration or wrong identification of Big ideas. Example: *Big ideas about measures and proportionality.*	Stating of Big ideas with elaboration. Example: *Idea on Equivalence. Students need to know why the fractions, even though written in different forms, are equivalent, and why they are equivalent.*
Are the verbal indicators of students' understanding identified and elaborated on?	No mention of verbal indicators.	Stating of verbal indicators or wrong identification of verbal indicators. Example: *Fraction of the circle, angle over 360, 2 pi r, pi r square*	Stating of verbal indicators with elaboration. Example: *Students can tell me that a fraction cannot be simplified because the terms are not a factor. E.g.* $\frac{2a+c}{2a-b} = \frac{c}{b}$ *is incorrect because 2a is not a factor.*

Data and Findings

- Analysis of lesson plans

The analysis of the lesson plans, both pre- and post-intervention, was guided by analytical prompts that were based on the prompts of the CTD Lesson Planning Tool. The lesson plans were examined for evidence related to the prompts and Tables 6, 7 and 8. show the outcomes for both teachers A and B. The juxtaposition of the pre- and post-intervention data for both teachers (A and B) in a table facilitated comparative analysis where necessary.

Table 6
The content focus of lessons

Analytical Prompt	Lesson Plan			
	Teacher A		Teacher B	
	Pre-I	Post-I	Pre-I	Post-I
Are the concepts that students will co-construct during the lesson identified and elaborated on?	0	2	0	2
Are the skills that students will acquire during the lesson identified and elaborated on?	1	2	1	2
Is the prior knowledge needed by the students for the lesson identified and elaborated on?	2	2	1	1
Are links to Big ideas considered in the planning?	0	2	0	1
Are links to future content in later stages of study considered in the planning?	0	2	0	0
Are links to other subject disciplines considered in the planning?	0	0	0	0
Are links to real life applications of the content considered in the planning?	0	0	0	1
Are the connections between content, prior knowledge and horizon content knowledge considered in the planning?	0	2	0	1

Legend: 2 – apparent, 1– partially apparent, 0 – not apparent

From Table 6, it is apparent that both Teachers A and B do not usually consider Big ideas, future content, links with other subject disciplines and real-life applications in their lesson planning. When prompted with the CTD Lesson Planning Tool, there were attempts to consider these in their revised lesson plans, although some aspects were still lacking in depth. It is also apparent that both Teachers A and B focused more on identifying the skills to be taught than the concepts to be developed in the lesson.

Table 7
The task focus of lessons

Analytical Prompt	Lesson Plan			
	Teacher A		Teacher B	
	Pre-I	Post-I	Pre-I	Post-I
Are learning tasks that help students understand the concepts identified?	2	2	2	2
Are practice tasks that help students develop the skills identified?	2	2	1	2
Is there an appropriate sequencing and scaffolding of the learning and practice tasks?	2	2	2	0
Are the profile of students considered in the planning?	0	2	0	2
Are the learning and instruction differentiated and elaborated on?	0	2	0	0

Are instructional strategies identified and elaborated on?	1	2	2	1
Are students' activity preference considered in the planning?	0	0	0	0
Are the plans to motivate students identified and elaborated on?	0	2	0	1

Legend: 2 – apparent, 1– partially apparent, 0 – not apparent

From Table 7, it is apparent that both teachers select, sequence and scaffold learning tasks and practice tasks very well during lesson planning. However, they do not usually consider the profile of their students, how to motivate them, what activities they like, and how to differentiate learning and instruction.

Table 8
The discourse focus of lessons

Analytical Prompt	Lesson Plan			
	Teacher A		Teacher B	
	Pre-I	Post-I	Pre-I	Post-I
Are the key questions or prompts identified and elaborated on?	1	2	2	2
Are important keywords or phrases used in the lesson identified and elaborated on?	0	1	1	1
Are students' responses anticipated and teacher's replies planned?	1	2	0	1
Are the verbal indicators of students' understanding identified and elaborated on?	0	2	0	1

Legend: 2 – apparent, 1– partially apparent, 0 – not apparent

From Table 8, it is apparent that both Teachers A and B usually plan for key questions, and are capable of anticipating the students' responses and how to reply to them when prompted to do so by the CTD Lesson Planning Tool. The tool also helped to challenge teachers to think beyond the usual assessments (practice and homework) and on assessing the students formatively through verbal indicators.

- Analysis of interview data

The audio recordings of the interviews were transcribed. In this section, text analysis of the transcripts and inferences drawn are presented. Table 9 shows the pre-interview prompts and responses of Teacher A and Teacher B.

Table 9
Pre-intervention interview prompts and data

Prompt:	Do you plan your mathematics lessons? If you do, how do you create your plan? Do you write out your plan?	Inferences
Teacher A	I **don't really write out a lesson plan**. Actually, **when I was using the textbook**, I didn't really plan for the lesson. I will just look through the textbook five minutes before the lesson and decide on the questions I was going to go through, and that's it. When I started to **create my own notes** for them, I have to plan through what I have to teach them... So in some sense, **I did plan for my lessons** while crafting my notes.	Teacher does not plan for lesson when he uses the textbook to teach but does plan when creating his own notes for the class; the notes

		function as his lesson plan.
Teacher B	**Yes, most of the time**. I will think through what are the **things I will need to cover** in the lesson, what are the **specific examples** I want to demonstrate and what I want the students to attempt, what are the **questions I want to ask**, that's about it. Most of the time it's just **in my head**.	Teacher plans for lesson but does not write out the plan.
Prompt	Do you use any template for your lesson plan? If yes, please elaborate.	
Teacher A	–	Teacher does not usually plan for lesson. The notes function as his lesson plan.
Teacher B	It's **one where the whole school uses**.	Teacher uses the lesson plan template provided by the school when required to submit a lesson plan for lesson observation.
Prompt	What are the key considerations guiding your plan? Elaborate as completely as possible.	
Teacher A	When I am starting a new chapter, then I will put in things that they need to know… bring the students back to how to do that in primary school, and tell the students that I will extend that knowledge to algebra, because they are the same idea. So I will try to **bring in prior knowledge**, if there is; if not, I will start with the simple basic examples. Actually some of the **examples** that I include in the notes are very similar to the textbook questions. I will judge whether the question from the textbook is simple to do, if it is then I will include the question. Along the way after more WAs (Weighted Assignments), I **realised that their standards dropped** and they don't understand what's going on. That's when I started to make sure the notes are not so intensive and taxing on them. I started off with the more basic things for them to settle first. If there's enough time, then I will move on to the difficult questions with them. I **already knew that their standard was quite good**. So from the very start, the lessons were fast paced, and there were more challenging questions because I know that they can cope. Homework questions were also slightly more challenging for them. I **try to think of fun ways to engage the students**, but actually I don't really have any ideas. So usually I will just follow the textbook.	When planning, the teacher focuses on prior knowledge and examples, and considers students' abilities and their interests. Teacher focuses on practice tasks, and gives homework related to the practice tasks as assessment.

	Usually in class we will **do the worked examples together**, then I will get them to **try the Try It questions themselves**… Then afterwards, at the end of the lesson, I will give them homework. **Homework will be very similar to what we have done in class**… The aim here is so that they have the time to revise what they have learnt for the lesson.	
Teacher B	I will first think about what **content I am going to cover**. Then look at what is the **lesson objective** and what is the **learning outcome**. Usually I will start with the outcome and I will **work backwards from there**, and ask myself what is it that the students need to achieve in this lesson. After that, I will think about **what are some of the activities** that we can do in this lesson to achieve that outcome.	When planning, teacher first considers the content, lesson objective and outcome. She then works backwards to think about the activities to include in the lesson.

It is apparent from the pre-intervention interview data in Table 9 that both Teachers A and B do plan for lessons, but do not typically write them out. When planning for their lessons, both Teachers A and B focus on identifying the tasks they were going to use for the students to learn the content they were going to teach. In addition, Teacher A aspires to consider the profile of the students and conduct fun activities to engage the students but is somewhat limited by his lack of experience and ideas.

Table 10 shows the post-intervention prompts and responses of Teacher A and Teacher B.

Table 10
Post-intervention prompts and data

Prompt:	Was the lesson planning tool helpful? Please elaborate.	Inferences
Teacher A	I think the lesson planning tool was **useful** for me because it is quite specific, to the point… frames for me **what I should put inside** this lesson planning tool… the old lesson plan template.. it's not very specific, so sometimes you **do not know what to fill in**. Or maybe as a beginning teacher, you might **miss out certain key things** that you are supposed to put in, such as the key questions that you should ask or the anticipated students' responses. This lesson planning tool is quite helpful because it really gives you an overview of what you are supposed to do… it makes us sequence the learning tasks and practice tasks that we are supposed to do. Let's say the new teacher is unsure of what to do and he has this lesson plan with him, he can **actually look at it and know that that is the sequence** that he's supposed to be doing. As compared to the old lesson template, it's very essay-like so sometimes you don't know what you are writing. If you are in class and you are stuck and you don't know what to do, you cannot stand there for 10 seconds reading through the lesson plan.	Teacher finds that the lesson planning tool is useful as it covers all the important aspects of a mathematics lesson. Teacher also finds that it is useful as it allows him to know the sequence of tasks at a glance.
Teacher B	The lesson planning tool **helped me to think through my entire lesson**, right from the "pre-lesson" such as what they need to know, the prerequisites, to how it will be extended later on. It does help me think through the lesson, and having to think through the key questions and prompts is **something that we don't do since back**	Teacher finds that the lesson planning tool is useful as it helps her to

	when we were practicum trainees. There's a reason why in practicum we do that, and there are good reasons to it. So it does help me think through and anticipate some of the possible responses, pre-empt them and think about how to answer some of these responses. So it definitely helps me to think through my lesson **more in-depth**. One of the things I did not think about in my previous lesson plan was Big ideas, so that prompt there **forces me to think** about what the Big idea is here, but even then, I am also not sure whether I got it totally right. The differentiating of concepts and skills took me a while to think about the exact difference. I think some teachers might not be able to see very clearly the difference between the concept and the skill. To them, a concept is a skill or a skill is a concept. So this part here **forces me to think** more about it and be clearer about the differences.	think through the whole lesson in-depth, and forces her to think of important aspects of a mathematics lesson that she used to do so as a trainee teacher but not anymore.
Prompt	Are there any aspect of the lesson planning tool which you find is lacking?	
Teacher A	From the perspective of a Beginning Teacher, sometimes I might over-plan or under-plan using this lesson planning tool… **there is a sequence but it does not tell me how long I should spend** on each section… Maybe one improvement could be to include a timeline or a space for the teacher to jot down how long he should be spending on each individual portion or section of the lesson plan.	Although the teacher finds the sequencing helpful, it can be improved by including a time element in the sequencing.
Teacher B	The template is very **cumbersome**. I have to keep referring to the prompts for every box just to know what exactly I have to write in each box. That was very confusing for me and made me very frustrated using the lesson planning tool. I prefer if the prompts were in a numbered list, and I just need to answer the prompts one by one. This **format was very annoying** for me, although I can see why the template was designed as such. From a user point of view, it's annoying as it can seem like it's all over the place. However, when I take a step back, I can understand why it was designed like that. Typing it on Microsoft Word was also very frustrating due to formatting troubles. Then again, it's the first time that I am using it. If I am using it for an entire year, then probably by the end of the year I will be very familiar with the tool and won't find it so cumbersome. I think one aspect that the lesson planning tool didn't focus on was on **blended learning**. This is a real struggle as my school is going to have a HBL (Home Based Learning) lesson once every fortnight, and when planning for the SOW (Scheme of Work), we need to factor that in. So when using the tool to plan for the entire unit, it will be good for the tool to be extended to help in preparing for the blended learning, such as identifying at which juncture of the unit can online learning take place.	Teacher finds that the lesson planning tool is too cumbersome to fill. Teacher also suggests to include blended learning, a new focus in the education system.
Prompt	What are the areas of your original lesson plan that you revised?	
Teacher A	No, I **never really revised anything** from my old lesson plan, other than it being **easier to fill up** now. It is because it's more succinct and I don't have to write it in an essay form. I could go straight to the point or write in point form. So, it's still the same lesson plan, just easier to write.	Teacher did not think that he revised anything.
Teacher B	**No I didn't make any changes**, just more of **reorganising** and filling in the relevant boxes accordingly.	Teacher did not think that she

		revised anything.
Prompt	Would you recommend this lesson planning tool to your colleagues? Please elaborate.	
Teacher A	I would recommend it to my colleagues because one good thing about it is that it is **easier to fill in**. The questions in the lesson planning tool are also more **straight to the point** and we really know what to fill in. Also, this lesson planning tool really looks into certain **issues that as teachers we sometimes overlook**. For example, I remember in this lesson planning tool, you asked us to think about the Big ideas in Mathematics and you even asked us how to do differentiation in our teaching. Maybe these are the things we usually overlook, even experienced teachers. With this tool, I think it really helps the teachers think more about their lessons.	Teacher finds that the lesson planning tool is concise and easy to fill, and helps him to focus him on the important aspects of a mathematics lesson.
Teacher B	I would introduce this to a **BT or someone who needs a structure to plan for a level or a topic that he/she has not taught before**. Or maybe someone who **needs help or guidance with the current new initiatives** such as Big ideas, blended learning, differentiated instruction, even though he/she might be experienced. Currently, we attend various courses to learn about all these new initiatives one by one, but there are **no opportunities to actually marry all these together and plan for a lesson that incorporates all these**. This lesson planning tool might help in marry all these new initiatives, although this might be too time consuming for them to do for every lesson.	Teacher finds that a teacher new to the teaching of a particular topic or initiative will benefit from the lesson planning tool.

From Table 10, it is apparent that both Teachers A and B found the CTD Lesson Planning Tool helpful as the tool prompted them to consider all the important aspects of mathematics lessons. In particular, Teacher B highlighted that the tool will be especially helpful for teachers who are new to the teaching of a particular topic or when incorporating new initiatives in the mathematics curriculum when planning lessons.

However, both Teachers A and B feel that the CTD Lesson Planning Tool can improve on its practicality. Teacher A hopes that there can be elements of time management of the lesson in the tool, while Teacher B noted that a prompt on blended learning was missing.

Discussion and CTD Lesson Planning Tool

The discussion is guided by two concerns. The first is "practice of teachers" with regards to planning of their mathematics lessons prior to the intervention, i.e. introduction to the CTD Lesson Planning Tool, and their "receptiveness of the tool" to reformulate their lesson plans. The second is inputs from the teachers for improvement of the tool with respect to "a teacher-friendly tool".

- Practice of teachers and receptiveness of the CTD Lesson Planning Tool

Both Teachers A and B did not consider many content, task and discourse aspects in their pre-intervention lesson plans. The lack of the teachers' consideration for these aspects was understandable as these aspects are not typically found in a mathematics lesson commonly guided by a template as shown in Lee (2009). However, when prompted by the CTD Lesson Planning Tool, teachers attempted to consider them, though with varying depths. It appears that the CTD Lesson Planning Tool provided teachers with a lens to reformulate their lesson plans incorporating aspects of a mathematically robust lesson.

In lesson plan guides such as those in Lee (2009), when teachers are prompted to state their Specific Instructional Objectives (SIOs), they tend to gravitate towards identifying student outcomes at the end of a lesson such as:

> *At the end of the lesson students should be able to find the arc length and area of sector given angle (in degrees) and radius of the circle.*

Therefore, it was expected in the pre-intervention lesson plan that teachers would focus more on delineating the skills rather than the concepts delved on during the lesson. The CTD Lesson Planning Tool helped both teachers to discern between the concepts and skills that their lesson will be based on. This helps to bring greater clarity of thought when selecting learning and practice tasks. This is a significant aspect of a mathematically robust lesson as tasks direct students to particular mathematical ideas (Kaur, 2010; Stein et al., 1996; Yeo, 2017).

An interesting observation from the data collected was that in their post-intervention interviews, both teachers did not think that they have revised their lesson plans, but the comparative analysis of their pre- and post-intervention lesson plans suggested otherwise. This might be attributed to the fact that what they think is important in a lesson plan are present in both. Both teachers highlighted in the pre-intervention interviews that they focused on identifying the tasks they were going to use when planning for their lessons. Task selection, sequencing and scaffolding were apparent in both their pre- and post-intervention lesson plans. It seems that lesson planning was viewed as a way to craft an end product – a practical guide for the lesson, rather than a process in which they go through in order to decide the best way to teach a specific content to a particular group of students. The lack of attention to the profile of their students, how to motivate them, what activities they like, and how to differentiate learning and instruction seems to support that. However, as Teacher B highlighted in her post-intervention interview, lesson planning is usually done "in (her) head". This might mean that many of the aspects that were not apparent in the pre-intervention lesson plans might have been considered by the teachers without them writing them down in their lesson plans. This finding supports that of Borko and Livingston (1989). The CTD Lesson Planning Tool simply forces the teachers to consider all these important aspects and pen them down, which was highlighted as the main reason why they found the tool helpful. Many of these aspects are aligned with the pedagogical practices in the STP (MOE, 2017a), and may guide teachers towards effective teaching and learning in their mathematics classrooms.

- <u>A teacher-friendly tool</u>

Both Teachers A and B found the CTD Lesson Planning Tool to be helpful. In particular, Teacher B felt that the tool will be especially helpful for teachers who are new to the teaching of a particular topic or when incorporating new initiatives in the mathematics curriculum when planning lessons. This might be the reason why Teacher A, a beginning teacher, felt that the tool was "easier to fill" and "to the point". As Teacher A was a novice and Teacher B an expert it is apparent that their lesson plans differed in scope bearing micro and macro perspectives respectively.

- <u>Revised CTD Lesson Planning Tool</u>

Although both teachers found the CTD Lesson Planning Tool helpful, they suggested ways the tool could be improved to enhance its practicality and "teacher-friendliness". Many teachers are busy and are unable to spend hours working on a lesson plan, which was the experience of both the teachers when using the tool. It was also suggested that the lesson planning tool should include elements of time management of the lesson and another prompt on blended learning.

These suggestions were taken into consideration, and the CTD Lesson Planning Tool was revised.

Firstly, the prompts were reorganised and similar prompts were grouped together in order to reduce the number of prompts without compromising on the robustness. Also, instead of grouping the prompts according to the categories of "fundamental considerations" and "deeper considerations", the prompts are now grouped according to the categories of "content considerations", "task considerations" and "discourse considerations".

Secondly, an additional prompt on blended learning (Prompt 11) was included. The aim of this prompt is to engage a teacher in thinking about how to introduce blended learning for the lesson or topic (Natarajan, 2021). It is important to note that blended learning is not simply just incorporating technology into learning. Blended learning combines e-learning with face-to-face teaching – students learn the basic content through e-learning before going deeper into the content with the teacher in class. This enables students to learn at their own pace, time and convenience. In order to ensure that blended learning is successful, students need to be motivated, self-disciplined and self-directed. Hence, blended learning should be planned deliberately and carefully in tandem with the other considerations.

Thirdly, teachers can now approach the prompts linearly, i.e. start with prompt 1, then 2 and so on. The template provided for teachers to fill their responses to the prompts was also revised to include a timeline of events that serves as a visual guide of the lesson for the teachers during the lesson.

Table 11 shows the prompts of the revised tool - A CTD Lesson Planning Tool for mathematics teachers.

Table 11
A CTD Lesson Planning Tool for mathematics teachers

Content Considerations *What am I teaching?* 1. What are the main concepts that students co-construct and skills that students acquire during the lesson/unit? 2. What prior knowledge is needed for students to understand the lesson/unit? 3. How is the content related to any Big Ideas, future content in later stages of study, other disciplines and/or real-life applications? 4. How can I demonstrate/make apparent/elicit the connections between the content, the prior knowledge and the horizon knowledge?
Task Considerations *What can I use for teaching and learning?* 5. What are some considerations I have regarding the profile of the students? 6. What kind of activities do the students prefer? How can I motivate the students? 7. What learning tasks can I use to get students to understand the mathematical concepts? 8. What practice tasks can I use to ensure students develop the mathematical skills? 9. How should I sequence and scaffold the learning tasks and practice tasks? 10. How can I differentiate the learning? 11. How can I incorporate blended learning?

Discourse Considerations

How can I facilitate learning?

12. What are some important keywords/phrases that students must understand?
13. What are some key questions/prompts I should pose? What are some anticipated students' responses and how do I respond?
14. What instructional strategies can I use to facilitate classroom discourse?
15. What are the verbal indicators to show that students have constructed the main concepts and acquired the skills?

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