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Beliefs of Pre-Service Mathematics Teachers in China About Mathematical Problem Solving - Focusing on Open / Close-Ended Problems

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Teachers' beliefs impact their practice in mathematics classrooms. This paper examines the beliefs of pre-service teachers in China related to teaching and learning mathematics, and open/close-ended problems during problem solving. Beliefs of 84 pre-service teachers in master's degree programme in three universities in China were measured using an adaptation of the Teachers' Beliefs on Their Practice (TBTP) instrument by Safrudiannur and Rott (2021). The results showed that beliefs of pre-service mathematics teachers in China differed for high ability and low ability classes. It appears that mathematical understanding is significantly less emphasised in open-ended problem solving than in close-ended problem solving, which raises concerns about potential risks. On the other hand, the close correlation between teacher beliefs about open-ended problem solving and the nature of mathematics may allow more affordance for pre-service teachers to put their own ideas into practice through open-ended problem solving.

Keywords: Teacher Belief, Open-ended Problem, Close-ended Problem, Mathematical Problem Solving, Pre-service Teacher

Introduction

Beliefs are "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Richardson, 1996, p. 103). Research has shown that generally mathematics teachers' instructional beliefs influence their instruction and student learning in mathematics classrooms (Philipp, 2007; Q. Zhang, 2022; Voss et al., 2013; Y. Wang et al., 2022). However, several comparative studies have shown that teachers' beliefs are influenced by social and cultural contexts (Cai, 2004; Philippou & Christou, 1998; Xenofontos & Andrews, 2014) and so teachers' beliefs are related to sociocultural factors. As mathematical problem solving pervades mathematics curricula worldwide, teacher beliefs related to problem solving in teaching and learning mathematics (e.g. Beswick, 2005) warrants attention when studying mathematics teacher beliefs (e.g. Xie & Cai, 2021).

An open-ended problem requires students to develop a method for answering the problem rather than finding an answer (Becker & Shimada, 1997). Open-ended problem solving has been recognised as beneficial in the attainment of higher cognitive objectives in the development of students' knowledge and skills (Becker & Shimada, 1997; Cifarelli & Cai, 2005). While previous research found that Chinese students performed better in process-constrained problems than in process-open problems (Cai, 2000), however, another recent replication study showed a significant improvement in both process-constrained and process-open tasks (Cai, 2022), which seems to demonstrate a measure of success in China's mathematics education reform in the last two decades.

A recent study in China found that teachers with less than 20 years of experience scored significantly lower in transmissive teaching belief than others (Y. Wang et al., 2022). In addition, another study found that Chinese pre-service teachers agreed more with constructivist teaching than traditional teaching (Yang et al., 2020). Both these studies show progress in preparation of pre-service mathematics teachers in China. Recently, the educational degree requirements of pre-service teachers in China have also levelled up with the development of the economy and society of the country (Huang et al., 2017). Secondary mathematics teacher preparation programmes in China place greater emphasis on mathematics content knowledge compared with pedagogical knowledge and student learning (Wu & Huang, 2018). Another concern regarding traditional teacher education in China is that it ignores the affective domains of teacher education (Q. Zhang, 2022).

Although much research has illustrated a connection between teachers' beliefs and practice (e.g., Rott, 2020; Q. Zhang, 2022); there appears to be a dearth specific to mathematics teacher beliefs related to mathematical problem solving (Harisman et al., 2019; Xenofontos & Andrews, 2014). This study attempts to contribute towards this dearth. The research questions that guide the exploration of the study involving pre-service mathematics teachers in China are:

1) What are pre-service mathematics teachers' beliefs specific to teaching and learning mathematical problem solving?
2) Do pre-service mathematics teachers' beliefs differ for open-ended and close-ended problem solving?
3) Is there a correlation between pre-service mathematics teachers' beliefs in mathematical problem solving and the nature of mathematics?

Literature Review

In this section, we first provide an operative definition of open-ended problem solving and briefly introduce previous findings on the affordances and constraints of open-ended problems with respect to student development and mathematics classrooms. Next, we briefly introduce teachers' belief in problem solving and current implementations of open-ended problems in Chinese mathematics curricula.

Open-ended problems

Non-routine problems may be open-ended or close-ended. Much of the research on problem solving involves close-ended non-routine problems; the solver knows that there is a goal but does not know how to reach it (Duncker, 1945). Nohda (2000) summarised the open-endedness of tasks into three different types: (1) the process is open (the problem has multiple correct ways to solve the original problem), b) the end products are open (the problem has multiple correct answers), and c) the ways to develop it are open (the problem could be developed by changing the conditions or attributes of the original problem). In the current study, the description of the open-ended task aligns with the former two characteristics: An open-ended task always has different correct answers and correct methods for the answers. With regard to open-endedness, the solver is free from confining constraints (such as prior experiences and biases) and may open his/her mind to better consider novel ideas and approaches (Sumara, 1996).

Research has illustrated that compared with close-ended problem solving, open-ended problem solving requires different abilities. Wang et al. (2022) suggested that cognitive abilities related to the generation of new information may play an important role in process-open problem solving, whereas cognitive abilities related to the retrieval of prior knowledge may be related to process-constrained problem solving. Compared to other types of tasks, creativity is most strongly associated with performance on open-ended non-routine problems (Schoevers et al., 2022).

As for teachers, their perspective towards open-ended problem-solving is also important in their instruction in mathematics classrooms. Pre-service and early career teachers were found to have stronger professional noticing when judging open-ended tasks (Yang et al., 2021). According to Sullivan et al. (2015), open-ended tasks require teachers to relinquish some level of control over student activity so that students have a chance to build their own mathematical arguments. Some researchers have elaborated on this argument by further discussing the affordances of open-ended problem solving (Chan & Clarke, 2017; Li et al., 2022; Nieminen et al., 2022; S. Zhang et al., 2022).

Open-ended problems have been widely initiated and implemented in Japan since the 1970s, and they were introduced into China in the 1990s (Zhang & Dai, 2004). One focus of mathematics curriculum reforms since the 21st century has been to include more real-life open-ended problems (Cai & Nie, 2007), even in national examinations.

Teachers' beliefs in problem solving

Ernest (1989a) categorised teacher beliefs about the nature of mathematics into the Instrumentalist view (mathematics is a useful but unrelated collection of facts, rules, and skills), the Platonist view (mathematics is a monolith, a static immutable product that is discovered, not created), and the Problem-solving view (mathematics is not a finished product, and its results remain open to revision). Beswick (2005) and Van Zoest et al. (1994) further extended teachers' beliefs in teaching and learning mathematics, which Safrudiannur and Rott (2021) followed in examining teaching and learning mathematical problem solving. Table 1 shows, Safrudiannur and Rott's conceptualisation of beliefs about the nature of mathematics and the teaching and learning of mathematics and problem solving in mathematics.

Table 1
The relationship between beliefs about the nature of mathematics and teaching and learning of mathematics and problem solving by Safrudiannur & Rott (2021)

Nature of Math	Teaching & Learning of Math		Teaching & Learning of MPS	
	Teaching	Learning	Teaching	Learning
Instrumentalist view	Teacher as an instructor	Students master skills correctly	Content-performance	Students follow and perform the teacher's methods correctly
Platonist view	Teacher as an explainer	Students understand conceptually	Content-understanding	Students follow and understand teacher's methods
Problem-solving view	Teacher as a facilitator	Students actively construct knowledge	Learner-interaction (student-centred)	Students create their own strategies

Although teacher beliefs may be categorised, Ernest (1989b) suggested that teachers may combine elements from more than one view, as evidenced by empirical studies (Liljedahl, 2009; Safrudiannur & Rott, 2021).

Background in China

In 1999, the State Council of China released an Action Plan to Revitalise Education in the Twenty-first Century that aimed to establish goals for education and to improve teachers' educational credentials. Since then, the number of normal schools and professional teacher colleges has decreased, whilst the number of normal universities and colleges increased in the following decade (Zhou, 2014). Master's programmes in mathematics education have been implemented in many provinces of China since 1998 (Huang et al., 2017). The enrolments in 2-year full-time professional Master of Education (MEd) programmes have further increased in recent years according to the Comments on the Implementation of the Excellence in Teacher Training Programme 2.0 (MOE, 2018). Such MEd programmes provide courses on curriculum and instruction, as well as a longer period of teaching practice (Law et al., 2018).

Method

Participants

The participants in the study are 84 (9 males and 75 female) students enrolled in master's degree programmes in three normal universities, A, B and C, in China. Universities A, B, and C are located in southern, north-eastern, and north-western China, respectively. The university admission criteria for University A are the most demanding whilst that for University C are the least. Their mean age is 23.45 (SD = 1.42) years and their mean teaching experience is 19.83 (SD = 36.59) weeks.

Table 2
Age and experience of teaching (weeks)

University	Participants		Age (years) Mean (SD)	Experience of teaching (weeks)
	Male	Female		
A	3	22	23.52 (0.87)	19.35 (16.09)
B	3	29	23.58 (1.82)	21.69 (54.24)
C	3	24	23.52 (1.34)	18.07 (22.80)
Total	9	75	23.45 (1.42)	19.23 (36.59)
	84			

Instrument

The instrument used for the study is adapted from the Teachers' Beliefs on Their Practice (TBTP), by Safrudiannur & Rott, 2021. It is similar to the TBTP in that it has the same 'rank-then-rate' items for three themes: Theme 1 - teaching and learning mathematics, Theme 2 - teaching and learning of problem solving and Theme 3 - the nature of mathematics. Each item has 3 statements: an Instrumentalist view statement, a Platonist view statement, and a Problem-solving view statement. The modifications in the adapted questionnaire were as follows:

1. Theme 3 was expanded to include parallel items for teaching of close-ended / open-ended problem solving (T - CPS / T - OPS) and learning of close-ended / open-ended problem solving (L - CPS/ L - CPS).
2. Every item in the questionnaire was repeated for high ability (HA) students and low ability (LA) students.

The authors translated the instrument to Chinese. Next, they added descriptions and examples of close-ended and open-ended problem solving (as shown in Figure 1).

Example for the Close-ended Problem
(The Academic Test for the Junior High School Students 2021 Beijing)
An enterprise has two production lines, A and B, which process the same raw material. In one day, production line A processes x ton of raw material in $(4x+1)$ hours: in one day, production line B processes y tons of raw material in $(2y+3)$ hours. On the first day, the enterprise assigns 5 tons of raw material to two production lines, A and B. Both production lines finish processing in one day, and the processing time is the same. The ratio of the number of tons assigned to production line A to the number of tons assigned to production line B is _______.

Example for Open-ended Problem
(A task from project-based learning)
QR codes are everywhere in our daily life. A QR code is a type of two-dimensional matrix barcode, using black and white squares to represent information. Please set a correspondence rule that can be used to represent a four-digit number by blacking out and leaving white space in the 4×4 square below and explain the validity of your correspondence rule.

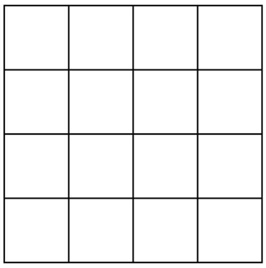

Figure 1. Examples of close-ended and open-ended problems in the instrument

Lastly, the authors trialled the questionnaire. They did this by inviting six pre-service teachers to complete the online questionnaire, following which they interviewed them about issues they encountered whilst doing the questionnaire. The feedback from the interviews helped the authors finalize the online questionnaire for the study.

Data collection and analysis

Data was collected in the summer of 2022 using an online questionnaire. Participants were invited to complete the questionnaire through a website or cell phone. Participants received a 10 RMB participation fee for completing the questionnaire. As in Safrudiannur and Rott (2021), the rating scores were analysed using SPSS 26.0 and R.

Results

To establish the reliability of the instrument Cronbach's alpha coefficient was first obtained for the three aspects (views) related to the nature of mathematics. The coefficients were 0.90, 0.92 and 0.86 for the Instrumentalist view, Platonist view, and Problem-solving view respectively. According to the criterion of Tavakol and Dennick (2011), as all Cronbach's alpha values were greater than 0.80, the instrument was reliable.

First, we analysed the pre-service teachers' rating scales for high ability (HA) and low ability (LA) classrooms. In high ability (HA) classes, teachers' beliefs were more associated with the problem-solving view, whereas in low ability (LA) classes, teachers' beliefs were more associated with the instrumentalist view. As shown in Table 3, Wilcoxon signed-rank tests found significant differences between teacher beliefs in HA and LA classes for teaching and learning mathematics, teaching and learning open-ended problem solving and teaching and learning close-ended problem solving.

Table 3

Comparison of teacher belief for High Ability and Low Ability classrooms

Items	Beliefs	HA Class	LA Class	Z	p
T-Math	Ins	5.37 (1.32)	6.24 (1.00)	5.208	<0.001**
	Pla	5.50 (1.24)	5.45 (1.36)	0.212	0.832
	Pro	5.96 (1.10)	4.29 (1.52)	6.505	<0.001**
L-Math	Ins	5.57 (1.29)	6.38 (0.93)	5.059	<0.001**
	Pla	5.69 (1.24)	5.51 (1.18)	1.419	0.156
	Pro	6.14 (1.00)	4.52 (1.40)	6.624	<0.001**
T-CPS	Ins	5.08 (1.38)	6.10 (1.15)	5.972	<0.001**
	Pla	5.67 (1.07)	5.88 (0.94)	1.665	0.096
	Pro	6.23 (0.92)	4.54 (1.35)	6.996	<0.001**
L-CPS	Ins	5.29 (1.27)	6.23 (0.96)	5.704	<0.001**
	Pla	5.43 (1.25)	5.56 (1.24)	0.885	0.376
	Pro	6.13 (1.15)	4.38 (1.46)	7.123	<0.001**
T-OPS	Ins	5.01 (1.44)	5.94 (1.17)	5.087	<0.001**
	Pla	5.42 (1.14)	5.39 (1.32)	0.007	0.994
	Pro	6.39 (0.85)	4.79 (1.40)	6.473	<0.001**
L-OPS	Ins	5.23 (1.39)	5.89 (1.19)	4.235	<0.001**
	Pla	5.48 (1.29)	5.43 (1.25)	0.373	0.709
	Pro	6.20 (1.02)	4.70 (1.53)	6.414	<0.001**

* significant for $p<0.05$; ** significant for $p<0.01$

Next, we compared the pre-service teachers rating scales by HA and LA classrooms together with teaching and learning mathematics, teaching and learning open-ended problem solving and teaching and learning close-ended problem solving with respect to the 3 views of the nature of mathematics (Instrumentalist, Platonist and Problem-solving). As shown in Table 4, using the Friedman test, it was found that teachers' rating scores varied between different views within HA and LA. According to the post hoc test, all differences between the Instrumentalist and Problem-solving views as well as the Platonist and Problem-solving views were significant. Regarding the comparison between the Instrumentalist and Platonist views, more significant differences occurred in the LA classes than in the HA classes.

Table 4

Comparison of pre-service teachers' rating scores by HA and LA, aspects of mathematical problem solving and the 3 views about the nature of mathematics

Items	Beliefs	Mean (Sd)	Friedman Test (df = 2)		Conover's post hoc test with Bonferroni correction (df = 166)		
			Chi-sqd	p	Between	t	p
T-Math in HA	Ins	5.37 (1.32)	13.189	0.001**	Ins vs. Pla	1.111	0.804
	Pla	5.50 (1.24)			Ins vs. Pro	3.556	0.001**
	Pro	5.96 (1.10)			Pla vs. Pro	2.445	0.047*
L-Math in HA	Ins	5.57 (1.29)	16.025	<0.001**	Ins vs. Pla	0.877	1.000
	Pla	5.69 (1.24)			Ins vs. Pro	3.830	0.001**
	Pro	6.14 (1.00)			Pla vs. Pro	2.953	0.011*
T-CPS in HA	Ins	5.08 (1.38)	46.131	<0.001**	Ins vs. Pla	3.267	0.004**

	Pla	5.67 (1.07)			Ins vs. Pro	6.806	<0.001**
	Pro	6.23 (0.92)			Pla vs. Pro	3.539	0.002**
L-CPS in HA	Ins	5.29 (1.27)	38.940	<0.001**	Ins vs. Pla	1.610	0.328
	Pla	5.43 (1.25)			Ins vs. Pro	6.038	<0.001**
	Pro	6.13 (1.15)			Pla vs. Pro	4.428	<0.001**
T-OPS in HA	Ins	5.01 (1.44)	64.889	<0.001**	Ins vs. Pla	2.321	0.065
	Pla	5.42 (1.14)			Ins vs. Pro	7.855	<0.001**
	Pro	6.39 (0.85)			Pla vs. Pro	5.535	<0.001**
L-OPS in HA	Ins	5.23 (1.39)	37.730	<0.001**	Ins vs. Pla	1.730	0.256
	Pla	5.48 (1.29)			Ins vs. Pro	5.985	<0.001**
	Pro	6.20 (1.02)			Pla vs. Pro	4.255	<0.001**
T-Math in LA	Ins	6.24 (1.00)	75.496	<0.001**	Ins vs. Pla	3.275	0.004**
	Pla	5.45 (1.36)			Ins vs. Pro	8.619	<0.001**
	Pro	4.29 (1.52)			Pla vs. Pro	5.344	<0.001**
L-Math in LA	Ins	6.38 (0.93)	77.821	<0.001**	Ins vs. Pla	4.419	<0.001**
	Pla	5.51 (1.18)			Ins vs. Pro	8.838	<0.001**
	Pro	4.52 (1.40)			Pla vs. Pro	4.419	<0.001**
T-CPS in LA	Ins	6.10 (1.15)	80.830	<0.001**	Ins vs. Pla	1.559	0.363
	Pla	5.88 (0.94)			Ins vs. Pro	8.463	<0.001**
	Pro	4.54 (1.35)			Pla vs. Pro	6.904	<0.001**
L-CPS in HA	Ins	6.23 (0.96)	85.508	<0.001**	Ins vs. Pla	3.251	0.004**
	Pla	5.56 (1.24)			Ins vs. Pro	9.138	<0.001**
	Pro	4.38 (1.46)			Pla vs. Pro	5.887	<0.001**
T-OPS in HA	Ins	5.94 (1.17)	41.862	<0.001**	Ins vs. Pla	2.631	0.028*
	Pla	5.39 (1.32)			Ins vs. Pro	6.445	<0.001**
	Pro	4.79 (1.40)			Pla vs. Pro	3.814	0.001**
L-OPS in HA	Ins	5.89 (1.19)	38.791	<0.001**	Ins vs. Pla	1.852	0.197
	Pla	5.43 (1.25)			Ins vs. Pro	6.086	<0.001**
	Pro	4.70 (1.53)			Pla vs. Pro	4.234	<0.001**

* significant for $p<0.05$; ** significant for $p<0.01$

Then we compared pre-service teachers' beliefs in open-ended and close-ended problem-solving using Wilcoxon signed-rank tests. The results are shown in Table 5. We also found the Spearman's rho correlation between teachers' beliefs about the nature of mathematics and their beliefs about teaching and learning mathematical problem-solving. More and stronger significant correlations were found for open-ended tasks.

Table 5
Comparison of pre-service teachers' belief about open-ended and close-ended mathematical problem solving

Teacher Belief about Nature of Math		HA				LA			
		T-CPS	L-CPS	T-OPS	L-OPS	T-CPS	L-CPS	T-OPS	L-OPS
Ins	Contents	0.45**	0.43**	0.54**	0.49**	0.51**	0.36**	0.43**	0.44**
	Truth	0.22*	0.26*	0.42**	0.36**	0.23*	0.20	0.30**	0.19
Pla	Contents	0.43**	0.51**	0.45**	0.53**	0.39**	0.31**	0.38**	0.39**
	Truth	0.36**	0.31**	0.43**	0.51**	0.42**	0.43**	0.58**	0.48**
Pro	Contents	0.37**	0.27*	0.36**	0.24*	0.26*	0.22*	0.25*	0.23*
	Truth	0.27*	0.31**	0.25*	0.42**	0.24*	0.22*	0.32**	0.36**

* significant for p<0.05; ** significant for p<0.01

Significant differences were found in the Platonist view for both HA (Z = 2.471, p = 0.013) and LA (Z = 3.771, p < 0.001) classes, indicating that teachers' rating scores on Platonist views (statements) were significantly higher in teaching close-ended than open-ended problem solving. Another significant difference was found in the teachers' rating scale on Instrumentalist views (statement) on learning problem solving in LA classes (Z = 2.817, p = 0.005), indicating that teachers rated mastery of close-ended tasks higher than open-ended tasks.

Lastly, we analysed the correlation between pre-service teachers' profiles (gender, experience, and university level) and beliefs using Spearman's rho test as shown in Table 6. No significant differences were found within the sample. This shows that the correlation between pre-service teachers' profiles and their beliefs on all aspects investigated in this study were not apparent.

Table 6
Correlation between pre-service teachers' profiles and instructional beliefs

Instrumentalist view	Math		CPS		OPS	
	HA	LA	HA	LA	HA	LA
Gender	-0.13	-0.15	-0.01	-0.15	-0.05	-0.07
Experience	0.05	0.08	-0.10	0.08	0.01	0.00
University level	-0.03	0.10	0.02	0.09	0.10	-0.01
Platonist view	Math		CPS		OPS	
	HA	LA	HA	LA	HA	LA
Gender	-0.15	0.13	-0.19	0.12	-0.21	0.01
Experience	0.01	-0.09	0.02	0.03	0.05	-0.04
University level	0.21	0.10	0.01	0.12	-0.05	0.03
Problem-solving view	Math		CPS		OPS	
	HA	LA	HA	LA	HA	LA
Gender	0.06	0.05	-0.18	-0.07	0.00	0.04
Experience	-0.04	-0.16	0.08	-0.08	0.02	-0.07
University level	0.14	-0.07	-0.10	-0.03	0.14	0.10

* significant for p<0.05; ** significant for p<0.01

Discussion

This study examined pre-service teachers' beliefs for high ability and low ability classes with respect to the teaching and learning of mathematics, teaching and learning close-ended problem solving and teaching and learning open-ended problem solving.

Safrudiannur and Rott (2021) found that students' abilities affected teachers' beliefs about the nature of mathematics, teaching/learning mathematics, and close-ended mathematical problem solving. Similar results were found regarding teaching/learning open-ended mathematical problem-solving in this study. Pre-service teachers tended to agree more with statements of problem-solving views in high-ability classes and with statements of instrumentalist views in low-ability classes. A further comparison of items in the same themes found many significant differences, illustrating that the TBTP is highly capable of distinguishing different beliefs in China.

Comparatively, we found that Chinese pre-service teachers' beliefs in teaching and learning mathematics differed from those in Indonesia, whose beliefs in HA classes were most consistent with the Platonist view (Pagiling et al., 2021; Safrudiannur et al., 2023; Safrudiannur, Labulan, et al., 2021), and similar results were found in terms of mathematical problem solving (Pagiling et al., 2021). Nevertheless, the results of this study on Chinese pre-service teachers' beliefs are similar to those of pre-service teachers in Germany and particularly similar to those of senior undergraduates (Safrudiannur, Belke, et al., 2021).

In this study regarding close-ended problem solving, pre-service teachers reported significantly less emphasis on understanding when teaching open-ended problem solving in all classes. Considering that mathematical understanding has continuously received attention in mathematics education (Cai & Ding, 2017), such weaknesses may entail potential risks. The case study of a pre-service teacher reported by Inoue and Buczynski (2011) found that she was not well prepared to consider students' unanticipated responses, which blocked the students' meaning-making attempts.

Past research has found that normal universities (involved in teacher education) with differing niches offer courses aimed at different goals (Law et al., 2018). The programmes of study of the students from the three universities involved in this study had four similar segments: general courses (11.1%–15.7% of all credits, English, 'political theories'), general education courses (21.1%–25% of all credits, 'philosophy of education', 'curriculum and instructional theories', 'methods of educational research', and 'psychological development and education'), professional education courses (42.1%–44.4% of all credits, concerning mathematics education, including textbooks, curricula, psychology, assessment, etc.), and practical training (21.1%–27.8% of all credits, including on-campus and off-campus practice). Compared with past programmes for undergraduates (Zhou, 2014), the proportion of professional education courses is markedly higher. Such similarities in the structures of programmes could possibly explain why no significant differences were found between pre-service teachers from different universities.

The results of the correlation analysis show that the open-ended problems probably afford teachers more opportunities to implement their own beliefs about the nature of mathematics, indicating that instruction about open-ended problem solving probably better allow teachers to implement their own beliefs about the nature of mathematics. Open-ended problems have already shown potential affordances of students' divergent thinking skills (Kwon et al., 2006), creative thinking (Fatah et al., 2016), and conceptual understanding (Yee, 2002); however, affordance is not equal to realisation, and sometimes it is more challenging for teachers (Chan & Clarke, 2017).

Limitations

Although we have provided vivid examples of open-ended and close-ended problems in the instrument, the beliefs reported by the pre-service teachers are more linked to espoused beliefs than to enacted beliefs (Ernest, 1989a). Two other limitations are that the respondents were recruited through convenience sampling, and the survey was conducted through an online questionnaire, both of which might cause reliability issues.

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