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# Subjective Visual Imageries in Probability: A Study on Students' Perspectives

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Living in this volatile, uncertain, complex, and ambiguous (VUCA) world necessitates the development of critical thinking and problem-solving skills among individuals. In this context, achieving fluency in statistics and probability plays a significant role. However, due to the highly abstract nature of the subject, particularly probability, students have difficulty understanding the concept and resort to solely relying on procedural approaches, resulting in an inability to develop the meaning of these objects. This study investigated the types of visual imageries formed by the students and how these imageries are utilized to attribute meaning to mathematical objects. There were 79 Grade 11 student participants from a public high school in Metro Manila, Philippines. Phenomenography was used to capture the different aspects and richness of their experiences. Results show that different perspectives have emerged when using the context of *teks*. These perspectives allow students to construct meanings for mathematical objects but require transitioning to a more structured visual representation.

Keywords: Probability, subjective visual imageries, *teks*, visualization objects

## Background of the Research Problem

Statistics and probability are crucial for making informed and evidence-based decisions. They play a vital role in various fields of study, including the social sciences, business and economics, science, and engineering, as well as medicine and public health. With a strong understanding of statistics and probability, individuals can effectively counteract misinformation, misinterpretation, and misrepresentation of data, especially in today's digital age where data is readily accessible from nearly everywhere, particularly in cyberspace (English & Watson, 2016; Gal, 2002). However, due to the highly abstract nature of Probability, students have predominantly relied on procedural approaches, resulting in a lack of concept retention rather than prioritizing conceptual approaches to learning probability (Garfield & Ahlgren, 1988; Shao, 2015). This reliance makes mathematical objects senseless entities, resulting in an inability to compare, categorize, visualize, and represent mathematical ideas (Davydov, 1990), which is evident from the poor performance of Filipino students in recent international assessments (Mullis et al., 2020; OECD, 2019).

Thom (1973) argued that the challenge does not lie in the complexity or rigor of mathematical knowledge but rather in the development of a "meaning" for mathematical objects within students' minds. Rivera (2010) posited that “children’s meaning-making processes are grounded on mental images or visual representations through shared practices of language and symbols" (pp. 43-44). In light of this, understanding the types of visual imagery formed by students and how these imageries are used to describe probabilistic terms are two important aspects considered in this paper.

## Research Objective

The main objective of this study was to investigate the types of visual imageries formed by the students and determine how these imageries are utilized to attribute meaning to mathematical objects. Specifically, the study sought to answer this question: What kinds of visual imagery (Presmeg, 1986) are provided by the three visualization objects (coins, dice, teks cards) presented?

## Conceptual Framework

The framework of this study was primarily concerned with the idea of providing students the opportunity to construct the meanings of mathematical objects and transforming their subjective visual imageries to more structured visual representations in understanding probability concepts. Guided by Rivera's (2010) notion of children's meaning-making processes, these existing mental images were categorized according to Presmeg's (1986) classification of visual imageries, which includes five distinct types: 1) concrete imagery (holistic mental images of everyday objects); 2) pattern imagery (visual-spatial representations of simple relationships); 3) memory imagery of formulae (formation of formulas mentally, akin to what is seen on a board or in a notebook); 4) kinesthetic imagery (images involving physical activities or muscle movements); and 5) dynamic imagery (active and evolving mental images).

## Research Methods

### *The participants*

The participants were Grade 11 students enrolled in various academic strands at a Senior High School: Science, Technology, Engineering, Mathematics (STEM); Humanities and Social Sciences (HUMSS); Accountancy, Business, and Management (ABM); and General Academic Strand (GAS). The researcher obtained ethics clearance to ensure participants consented to the study and were well-aware of the conditions of their participation.

### *Research instrument*

A research-made written test, the Test on the Basic Probability Concepts (TBPC), was developed to identify the existing types of visual imagery used by the students to solve the given probability problems with increasing difficulty. The first part of TBPC requires students to find the set of all possible outcomes of an experiment. Only one out of the three experiments (i.e., tossing coins, rolling dice, and flicking *teks* cards) shall be chosen by the students to answer. The second part of the TBPC further examines the visualization object which allows the students to elicit multiple types of visual imageries.

### *Method of data collection*

Phenomenography (Kinnunen & Simon, 2012) was used to comprehensively investigate the various facets and depth of the visual imageries that students used in solving problems involving probability. This exploratory research was structured based on the principles of design science and consisted of two main phases. In the first phase, the researchers administered the Test on the Basic Probability Concepts (TBPC) to the participating students. This test was validated and considered sufficient for the purpose of the study by two mathematics education experts and one mathematics expert. In this test, three different problems which use three different visualization objects were introduced to the students. Two of the visualization objects presented are conventionally used in the classroom (i.e., coins and dice) while the third one is an alternative visualization object that is not used in the classroom but is like the context of tossing a coin, which is the traditional Filipino game played by most students, *teks*. Teks cards are cards that contain comic strips and are collected by playing with other students. These cards contain comic strips from famous movies, TV series, and novels.


*Figure 1.* Physical Structure of a teks card

As shown in Figure 1, based on its physical structure, a teks card also has two faces which is similar to the heads and tails of a coin. The winning side is the front face with a picture, called *Tao* or *Cha*, while the losing side is the back face without a picture, called *Ibon* or *Chob*.

The mechanics of *teks* are as follows. Each player gives one card as their entry into the game (*pamato*). Then all cards from each player will be flicked simultaneously. To win the game, one's card must have a front face (F) outcome. If many cards end up facing front, the players flick the remaining cards again until only one card showing the front face is left.

Rich and valid data were also collected by interviewing each participant to describe, in their own words, the types of visual imagery they use and how they experience it. Following the administration of the test, the analysis of students' responses to each test item and a comprehensive review of interview transcripts were conducted to establish distinct categories of visual imageries. To ensure the credibility and accuracy of the emerging categories of visual imageries, a rigorous process was undertaken involving three separate readings.

### *Data analysis*

Following the phenomenography data analysis process (Kinnunen & Simon, 2012), three readings were done to ensure the reliability and validity of the list of emerging categories of visual imageries. In the first reading, the researcher first familiarized himself with the students' subjective visual imageries and identified the initial set of emerging themes by examining the students' solutions to each item and by reading the interview transcripts. Similarly, in the second reading, two experts (mathematics education and mathematics) repeated the process done by the researcher in the first reading. In the third reading, after a long and careful discussion, the identified list of emerging themes was concertedly refined and redefined by the researcher and the two experts to come up with the final set of categories.

## Results and Discussion

The first part of the TBPC required students to find the set of all possible outcomes of an experiment they themselves have chosen between the three contexts provided: (1) tossing three coins, (2) rolling three dice, or (3) flicking three teks cards. The results show that students' preliminary actions on perceived objects could vary from one context to another.

Table 1 presents the varying actions of students on perceived objects (i.e., the concept of sample space) according to their prior knowledge, understanding of outcomes, existing visual imageries, and background experience.

Table 1.
*Students' Pre-Action Levels and Corresponding Actions on Perceived Objects*

| **Pre-Action Level** | **Students' Actions on Perceived Objects** | | |
|---|---|---|---|
| | **Coins** | **Dice** | **Teks** |
| Prior Knowledge | Students solved the problem using concepts that are only limited to recalling a tree diagram or the formula (2^n). They cannot relate the concept of a sample space to previously learned concepts and, hence, treated each concept independently. | Students solved the problem by making multiple representations such as dots, numbers, and the number line. They cannot relate the concept of a sample space to previously learned concepts and, hence, they resorted to concretely visualizing the outcomes of rolling a dice. | Students solved the problem using a more descriptive method of listing down the outcomes based on the scenarios of the game teks. Oftentimes, they reflect on their experiences playing teks rather than just recall and solely rely on procedural approaches. |
| Understanding of Outcomes | Students define an outcome as a complicated concept which requires a more sophisticated method and formal mathematics (diagrams or formula). The arrangement of the sequence of the outcome is problematic. | Students define an outcome as a concept which oftentimes requires a more formal method (using diagrams or formula). Their description of an outcome depends on the concrete image they used to represent it. | Students define an outcome as a non-formal mathematical term that simply describes the result or consequence of playing teks. The arrangement of the sequence of the outcome is not problematic. |
| Existing Visual Imageries | Students solely rely on memory imagery of formulae in solving the problem. | Students solely rely on concrete imagery (e.g., diagrams, dots, boxes, number line) in solving the problem. | Students combine multiple types of visual imageries (e.g., kinesthetic, dynamic, concrete and pattern, dynamic and memory) in solving the problem. |

| | | | |
|---|---|---|---|
| Background Experience | Students do not use the physical characteristics of the coin (having a head on one side and a tail on the other). Instead, when students see the coin, they resorted to using the tree diagram. Hence, their understanding of the outcome is not directly associated with the physical object of the coin. The Head and Tail of the coins are represented as H and T respectively and students' reliance on the tree diagram and formula is the only way to manipulate these objects. | Students use the physical characteristics of the dice to determine the set of all possible outcomes. Hence, their understanding of the outcome is directly associated with the physical object of the dice. However, they cannot correctly represent the dice on paper, so they need to construct more concrete images and diagrams to manipulate the different faces of the dice (represented as 1, 2, 3, 4, 5, 6). | Students use the mechanics of the teks as they recognize it as their childhood game; hence, their description of the outcomes is simply based on their experience of playing teks. |

Among the three contexts presented, results show that *teks* provides a more descriptive perspective to students and allows them to use multiple types of imageries, while the two other contexts (coins and dice) limit students' visual imageries to just one type of imagery and impede their ability to transition into other imageries. Physical characteristics of the visualization objects were not used by the students who solved the coin problem, whereas these were used by those who solved the dice problem and the *teks* problem. However, the outcomes of the dice problem could not be correctly represented by the students on paper leading them to construct more concrete images and diagrams to manipulate the different faces of the dice. For students who solved the *teks* problem, they used the mechanics of the *teks* as they recognize it as their childhood game; hence, their description of the outcomes is simply based on their experience of playing *teks*. Considering these results, an in-depth analysis of the types of visual imagery elicited and employed when solving the *teks* problem was conducted by having all students solve the *teks* problem.

Figure 2 illustrates students' concrete representation of the front and back faces of the *teks*. While the mechanics of the game may vary over time as it has evolved, the fundamental rule of elimination remains constant: the front face of the *teks* represents the winning side, and the back face signifies the losing side. However, intriguingly, in this particular form of imagery, students hold the belief that an entry card or "*pamato*" is selected by some inexplicable ad hoc force, suggesting that certain cards are luckier than others. Hence, due to these beliefs, different perspectives have further emerged. These are the following: F–Perspective, B–Perspective, All Winner/Loser Perspective, One Winner Perspective, One Loser Perspective, Equiprobability Perspective, Varied Outcome Perspective, Defined Extreme but Varied Outcome Perspective, Outcome–Focused Perspective.

1. The F–Perspective. This is when students expect that for every round, everyone will most likely get a front face (F). Each one of them believes that they have chosen the best teks card to win the game.
2. The B–Perspective. This is when students expect that for every round, everyone will most likely get a back face (B). Each one of them believes that they have chosen the

best teks card to win the game; however, the "ad hoc force" cannot decide which teks card should win making them all lose instead.

3. All Winner/Loser Perspective. This is when students expect that for every round, everyone will most likely get a front face (F) or a back face (B). Each one of them believes that they have chosen the best teks card to win the game, but the "ad hoc force" cannot decide which teks card should win or lose.
4. One Winner Perspective. This is when students expect that for every round, there is only one player that is chosen by the "ad hoc force" to win.
5. One Loser Perspective. This is when students expect that there is a loser (i.e., to be eliminated) every round until only one winner emerges.
6. Equiprobability Perspective. This is when students expect that for every round, each one of them has an equal chance to be selected by the "ad hoc force" to win the game.
7. Varied Outcome Perspective. This is when students think that they cannot predict which teks cards are chosen by the "ad hoc force" to be the luckiest among the rest of the teks cards.
8. Defined Extreme but Varied Outcome Perspective. This is when students think that the chance of getting all front faces, or all back faces is relatively less likely to happen. However, they cannot predict which teks cards are chosen by the "ad hoc force" to be the luckiest among the rest of the teks cards.
9. Outcome–Focused Perspective. This is when students focus only on a particular outcome. They believe that by looking at the front face of each entry card (pamato) as well as the number of times it wins, they can predict the outcome of the following rounds.

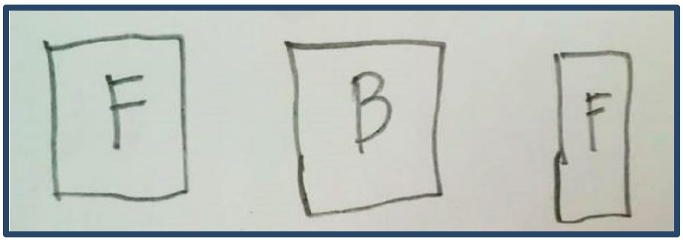


*Figure 2.* Concrete imagery

The second type of imagery is pattern imagery. In Figure 3, a student concretely represented the three teks cards as "B-F-B" and wrote several random F and B symbols around the concrete image "B-F-B". Based on the interview conducted, the student narrates that writing this down appears to be the best way to explicitly see and determine other possible combinations and patterns that might emerge.

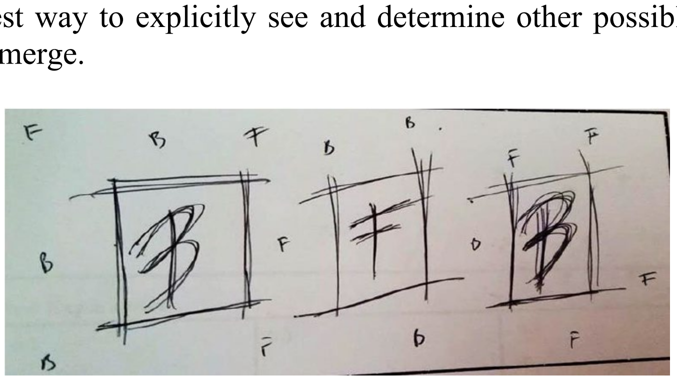


*Figure 3.* Pattern imagery

When students realized that the teks problem paralleled the traditional coin problem, they tended to recall the formula or the process for solving probability problems related to coins. This suggests that students utilized the "Memory Image of Formulae" visual imagery, as shown in Figure 4. Based on the interview, most of them mentioned that recalling the formula was the easiest way to answer the problem; hence, they get stuck on the problem or, worse, leave the items blank when they failed to remember the formula.

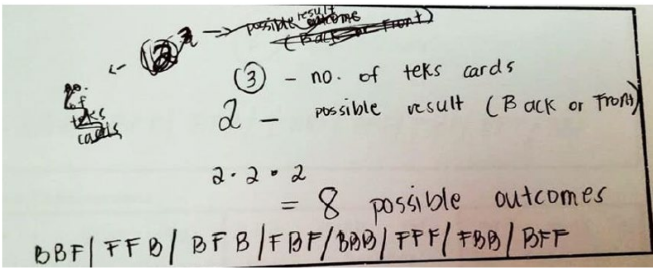


*Figure 4.* Memory Imagery of Formulae (MIF)

As shown in Figure 5, students who used the Kinesthetic Imagery resourcefully devised their own teks cards and started flicking them in the air to list the outcomes. This was done to recall or perceive the actual experience of playing teks. Some of the students were also observed to be flicking an imaginary set of teks cards in the air while reading the question again and again. This type of imagery is often combined with concrete and dynamic imageries.

*Figure 5.* Kinesthetic Imagery (KI)

The last type of visual imagery is dynamic imagery, shown in Figure 6, wherein an active image of the teks cards emerges in their mind since they believe that the set of all possible outcomes depend on the movements of the teks cards and some external factors affecting these movements. They observe or imagine these movements and think of a possible explanation why a certain outcome occurs.

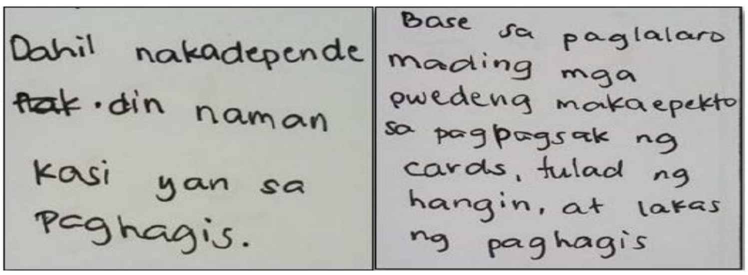


*Figure 6.* Dynamic Imagery (DI)

## Conclusion

Among the three contexts presented, results showed that teks provides a more descriptive perspective to students and allowed them to use multiple types of visual imageries, while the other two contexts (coins and dice) limit students' visual imageries to just one type and impede their ability to transition into other imageries. With the right visualization object, students were able to interpret, reflect, and reconfigure rather than just recall and rely solely on procedural approaches. However, these perspectives need to be transformed into a more structured visualization through design science.

Some argue that dealing with these subjective perspectives inside the classroom is challenging. Nevertheless, the concept of subjective visual imagery provides students with the opportunity to construct meanings for mathematical objects. This would hopefully encourage teachers to consider the students' subjective visual imageries since it is an essential part of reaching the level of mathematical abstraction. This could also help in providing a more comprehensive and progressive view of visualization that is not limited to pictures and diagrams but also as a product of shared culture and experiences among the students.

Further studies are recommended to explore the nature and characteristics of students' visual imagery and how these should be incorporated into the design of learning materials.

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